ECE 588 – Electricity Resource Planning

9. Multi–State Unit Model

George Gross
Department of Electrical and Computer Engineering
University of Illinois at Urbana–Champaign
The two-state representation of the availability of a unit results in a model with many limitations:

- inability to represent states of the system with less than full capacity
- reduced fidelity in the simulation of the unit available capacity characteristics
MULTI-STATE UNIT CAPACITY AVAILABILITY MODEL

- induced necessity to reduce available capacity information to the two-state representation, even for units with multi-state operations

- For simulation with higher fidelity, we need more detailed available capacity models to represent generation unit availability characteristics.
THREE-STATE MODEL OF UNIT $i$

Available Capacity

$$s_i = 1 - q_i - r_i \text{ (full availability rate)}$$

- $c_i$: Total availability (rated capacity)
- $d_i$: Partial outage state (derated capacity)
- $0$: Forced outage state

$q_i$: F.O.R. (forced outage rate)
$r_i$: P.F.O.R. (partial forced outage rate)
THREE-STATE MODEL OF UNIT $i$

$F_{A_i}(x)$

$p_i = r_i + s_i$

$F_{A_i}(x)$

$s_i$

$q_i + r_i$

$q_i$

$r_i$

$s_i$

$q_i$

$0$

$d_i$

$c_i$

$x$

$1$

$f_{A_i}(x)$

$0$

$d_i$

$c_i$

$x$
Consider the capacity outage r.v.

\[ Z_i = c_i - A_i \]

The density function and the c.d.f. are

\[ f_{Z_i}(x) = f_{A_i}(-[x - c_i]) \]

\[ P\{Z_i \leq x\} = 1 - P\{Z_i > x\} \]

\[ = 1 - P\{-Z_i < -x\} = 1 - P\{A_i - c_i < -x\} \]

\[ = 1 - P\{A_i < c_i - x\} = 1 - F_{A_i}[-(x - c_i)] \]
The representation of the unit loading uses the convolution iteratively

\[ L_i = Z_i + L_{i-1} \quad \text{with} \quad L_0 = L \]

The three–state availability results in the following convolution expression

\[ \mathcal{L}_i(x) = s_i \mathcal{L}_{i-1}(x) + r_i \mathcal{L}_{i-1}(x - [c_i - d_i]) + q_i \mathcal{L}_{i-1}(x - c_i) \]
since \[ s_i + r_i + q_i = 1 \]

we introduce the notation

\[ p_i = s_i + r_i \]

and

\[ q_i = 1 - p_i \]

The expression for expected energy production is

\[
\mathcal{E}_i = T \left\{ \begin{array}{l}
p_i \int_{C_{i-1}}^{C_i} \mathcal{L}_{i-1}(x) dx + s_i \int_{C_{i-1}}^{C_i+d_i} \mathcal{L}_{i-1}(x) dx \\
C_{i-1} + d_i \int_{C_{i-1}}^{C_i} \mathcal{L}_{i-1}(x) dx + C_i \int_{C_{i-1}}^{C_i+d_i} \mathcal{L}_{i-1}(x) dx \\
\end{array} \right\}
\]
\[ i = i_{-1} - L \cdot C_{i_{-1}} + d_i - \text{area multiplied by } p_i \]
\[ \mathcal{L}_{i_{-1}}(\cdot) \]

\[ i = i_{-1} + s_i - \text{area multiplied by } s_i \]

\[ C_{i_{-1}} \leq x \leq C_i \]
Unit 1 is modeled as a 3-state unit with

<table>
<thead>
<tr>
<th>state</th>
<th>capacity MW</th>
<th>probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>forced outage</td>
<td>0</td>
<td>$q_1 = 0.1$</td>
</tr>
<tr>
<td>partial outage</td>
<td>$d_1 = 300$</td>
<td>$r_1 = 0.2$</td>
</tr>
<tr>
<td>zero outage</td>
<td>$c_1 = 500$</td>
<td>$s_1 = 0.7$</td>
</tr>
</tbody>
</table>
EXAMPLE: DATA

- The \( LDC \) model has a quadrilateral shape with a base load of 600 MW and a peak load of 1300 MW with a straight line joining the peak and the base load points.

- We are told that this \( LDC \) is constructed from the chronological load of a period of 1,000 h.
EXAMPLE: \textit{LDC}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{example_ldc}
\caption{Example of Load Duration Curve (LDC).}
\end{figure}
EXAMPLE: INVERTED $LDC$

\[ L_0(x) = L(x) \]
EXAMPLE: UNIT 1 STATES

\[ s_1 = P \{ A_1 = c_1 \} = 0.7 \]
\[ r_1 = P \{ A_1 = d_1 \} = 0.2 \]
\[ q_1 = P \{ A_1 = 0 \} = 0.1 \]
EXAMPLE: LOADING OF UNIT 1

\[ L_1 = L_0 + (c_1 - A_1) \]

\[ \mathcal{L}_1(x) = s_1 \mathcal{L}(x) + r_1 \mathcal{L}\left[ x - (c_1 - d_1) \right] + q_1 \mathcal{L}\left( x - c_1 \right) \]
EXAMPLE: LOADING OF UNIT 1

\[ 0.7 \mathcal{L}(x) \]

\[ 0.2 \mathcal{L}(x - 200) \]

\[ 0.1 \mathcal{L}(x - 500) \]
IELDC $\mathcal{L}_1(g)$
MULTI-STATE UNITS

- The representation of units by multiple-state models may be extended to any number of states for which data are available.

- The simulation of the loading of multiple-state units can be accommodated in the production simulation but involves the costs of the additional complexity in the representation.