8. Expected Costs of Generation
ALTERNATIVE EVALUATION OF $\mathcal{E}_k$

- We define the mixed-type r.v. $P_k$ to denote the power demanded from unit $k$

$$P_k = \begin{cases} 
  c_k & \text{if } L_{k-1} \geq C_k \\
  L_{k-1} - C_{k-1} & \text{if } C_{k-1} \leq L_{k-1} < C_k \\
  0 & \text{if } C_{k-1} > L_{k-1} 
\end{cases}$$
ALTERNATIVE EVALUATION OF $\mathcal{E}_k$

- We construct the c.d.f. of $P_{\sim_k}$

$$F_{P_{\sim_k}}(x) = \begin{cases} 
1 & x \geq c_k \\
F_{L_{k-1}} \left[ x + C_{k-1} \right] & 0 \leq x < c_k \\
0 & x < 0 \end{cases}$$

and its density function
ALTERNATIVE EVALUATION OF $\mathcal{E}_k$

\[
f_{p_k}(x) = \begin{cases} 
0 & x > c_k \\
\left\{1 - F_{L_{k-1}}\left[x + C_{k-1}\right]\right\}\delta(x - c_k) & x = c_k \\
f_{L_{k-1}}\left[x + C_{k-1}\right] & 0 < x < c_k \\
F_{L_{k-1}}\left[x + C_{k-1}\right]\delta(x) & x = 0 \\
0 & x < 0 
\end{cases}
\]
ALTERNATIVE EVALUATION OF $\tilde{\varepsilon}_k$

$$F_{P_{\sim k}}(x)$$

0  \quad C_k  \quad MW

$F_{L_{k-1}}[x + C_{k-1}]$
ALTERNATIVE EVALUATION OF $\mathcal{E}_k$

The evaluation of $\mathcal{E}_k$ is simply

$$\mathcal{E}_k = T \left[ E \left\{ P_{\sim k} \mid A_k = c_k \right\} p_k + E \left\{ P_{\sim k} \mid A_k = 0 \right\} \left(1 - p_k\right) \right]$$

$$= p_k T E \left\{ P_{\sim k} \mid A_k = c_k \right\}$$

We have that

$$E \left\{ P_{\sim k} \mid A_k = c_k \right\} = E \left\{ P_{\sim k} \right\} = \int_{-\infty}^{\infty} x f_{P_{\sim k}}(x) \, dx$$
ALTERNATIVE EVALUATION OF $\tilde{\varepsilon}_k$

$$\lim_{\varepsilon \to 0} \int_{0-\varepsilon}^{c_k + \varepsilon} x f_{\mathcal{P}_{\approx k}^x} (x) \, dx$$

$$= c_k \left( 1 - \int \left[ c_k + C_{k-1} \right] \right) + \lim_{\varepsilon \to 0} \int_{0 + \varepsilon}^{c_k - \varepsilon} x f_{\mathcal{P}_{\approx k}^x} (x) \, dx$$

$$= c_k \left( 1 - \int \left[ C_k \right] \right) + \lim_{\varepsilon \to 0} x F_{\mathcal{P}_{\approx k}^x} (x) \bigg|_{0 + \varepsilon}^{c_k - \varepsilon} - \int_{0 + \varepsilon}^{c_k - \varepsilon} F_{\mathcal{P}_{\approx k}^x} (x) \, dx$$
ALTERNATIVE EVALUATION OF $E_k$

\[
E_k = c_k \left\{ 1 - F_{L_{k-1}} \left[ C_k \right] \right\} + c_k F_{L_{k-1}} \left[ C_k \right] - \lim_{\varepsilon \to 0} \int_{0 + \varepsilon}^{c_k - \varepsilon} F_{P_{k}} (x) \, dx
\]

\[
= c_k - \int_{0}^{c_k} F_{L_{k-1}} \left[ x + C_{k-1} \right] \, dx
\]

\[
= \int_{0}^{c_k} \left\{ 1 - F_{L_{k-1}} \left[ x + C_{k-1} \right] \right\} \, dx
\]

\[
= \int_{C_{k-1}}^{C_k} L_{k-1} (x) \, dx
\]
Therefore,

\[ \mathcal{E}_k = p_k T \int_{C_{k-1}}^{C_k} \mathcal{L}_{k-1}(x) \, dx, \]

which is identical to the previous result.

This representation using the physically more intuitive \( P_k \) r.v. provides some additional insights into the evaluation of \( \mathcal{E}_k \).
GENERATION UNIT ECONOMICS

$w(x)$

$C_{\min}$ to $C_{\max}$

MW

$/h$
The interpretation of the unit economics curve is the following: \( w(x) \) gives the production costs when the unit is loaded at \( x \) MW capacity.

Then,

\[
w'(x) = \frac{dw(x)}{dx} \bigg|_x
\]

gives the \textit{marginal costs} of energy when the unit is loaded at \( x \) MW.
Engineers typically refer to \( w'(x) \) as the \textit{incremental or decremental costs}. 

We have

\[
w(x) = w(c^{min}) + \int_{c^{min}}^{x} w'(x) \, dx, \quad c^{min} \leq x \leq c^{max}
\]

Note that in terms of the heat rate \( h(\cdot) \)

\[
w(c^{min}) = h(c^{min}) c^{min}
\]
GENERATION UNIT ECONOMICS

$\$/h

$w(c_{\text{min}})$

$\theta$ slope

$c_{\text{min}}$  $c_{\text{max}}$

MW

ECE 588  © 2002 - 2016 George Gross, University of Illinois at Urbana-Champaign, All Rights Reserved.
GENERATION UNIT ECONOMICS

- An operating unit must be loaded with capacity value specified.
- The capacity must be greater or equal to $c^{\text{min}}$.
- The economics of the unit for $0 \leq x < c^{\text{min}}$ are not defined since a unit cannot operate in that region.

- We refer to the costs of operating a unit at $x = 0$ as the no load costs.

ECE 588 © 2002 - 2016 George Gross, University of Illinois at Urbana-Champaign, All Rights Reserved.
EVALUATION OF EXPECTED GENERATION COSTS

- We define for mathematical purposes only

\[ w(0^+) = w(c_{\text{min}}) \]

and

\[ w'(x) = 0 \quad 0 \leq x < c_{\text{min}} \]

- We use the alternative evaluation of \( \mathcal{E}_k \) to compute the expected value of the power generation costs of the unit \( k \)
EVALUATION OF EXPECTED GENERATION COSTS

- We first define

$$ W_k \triangleq w_k \left( P_k \right) $$

and compute

$$ E \left\{ W_k \left| A_k = c_k \right. \right\} = E \left\{ W_k \right\} $$

$$ = \int_{-\infty}^{\infty} w_k(x) f_{P_k}(x) dx $$

$$ = \lim_{\varepsilon \to 0} \int_{0-\varepsilon}^{c_k+\varepsilon} w_k(x) f_{P_k}(x) dx $$
EVALUATION OF EXPECTED GENERATION COSTS

\[ w_k(c_k) \left\{ 1 - F_{\bar{Z}_{k-1}} \left[ c_k + C_{k-1} \right] \right\} + \]

\[ \lim_{\varepsilon \to 0} \int_{0}^{c_k - \varepsilon} w_k(x) f_{P_k}(x) \, dx \]

\[ = (A) + \lim_{\varepsilon \to 0} \left\{ w_k(c_k - \varepsilon) F_{P_k}(c_k - \varepsilon) - \\ w_k(0 + \varepsilon) F_{\bar{P}_k}(0 + \varepsilon) \right\} - \lim_{\varepsilon \to 0} \int_{0}^{c_k - \varepsilon} w_k'(x) F_{\bar{P}_k}(x) \, dx \]
EVALUATION OF EXPECTED GENERATION COSTS

\[
\begin{align*}
\mathcal{A} & + w_k(c_k) F_{L_{k-1}} \left[ C_k \right] - w_k(0^+) F_{L_{k-1}} \left[ C_{k-1} \right] - \\
& \lim_{\varepsilon \to 0} \int_{0^+}^{c_k - \varepsilon} w_k'(x) F_{L_{k-1}} \left[ x + C_{k-1} \right] dx \\
& = w_k(c_k) F_{L_{k-1}} \left[ C_{k-1} \right] - w_k(0^+) F_{L_{k-1}} \left[ C_{k-1} \right] - \\
& \int_0^{c_k} w_k'(x) F_{L_{k-1}} \left[ x + C_{k-1} \right] dx
\end{align*}
\]
EVALUATION OF EXPECTED GENERATION COSTS

\[
= w_k (0^+) + \int_0^{c_k} w'_k (x) \, dx - w_k (0^+) F_{L_{k-1}} [C_{k-1}] - \\
\int_0^{c_k} w'_k (x) F_{L_{k-1}} [x + C_{k-1}] \, dx
\]

\[
= w_k (0^+) \left\{ 1 - F_{L_{k-1}} [C_{k-1}] \right\} + \\
\int_0^{c_k} w'_k (x) \left\{ 1 - F_{L_{k-1}} [x + C_{k-1}] \right\} \, dx
\]
EVALUATION OF EXPECTED GENERATION COSTS

\[ E \{ W_k \} = w_k \left( 0^+ \right) \mathcal{L}_{k-1} [ C_{k-1} ] + \]

\[ \int_{C_{k-1}}^{C_k} w' \left[ x - C_{k-1} \right] \mathcal{L}_{k-1} (x) \, dx \]

- We introduce the notation

\[ C_k = \text{expected generation costs of unit } k \]

\[ = T \left\{ E \left( W_k \mid A_k = c_k \right) p_k \right\} \]
EVALUATION OF EXPECTED GENERATION COSTS

Therefore,

\[ C_k = p_k T \left\{ w_k (\theta^+) \mathcal{L}_{k-1} [C_{k-1}] + \right. \]

\[ \int_{C_{k-1}}^{C_k} w'_k \left[ x - C_{k-1} \right] \mathcal{L}_{k-1} (x) \, dx \}\]

Consider the costs \( C_k \) as a function of the loaded capacity \( x \), with \( C_k \geq x \geq c_k^{min} \)
EVALUATION OF EXPECTED GENERATION COSTS

\[ C_k(\mu) = p_k T \left\{ w_k(0^+) \mathcal{L}_{k-1} \left[ C_{k-1} \right] + \right. \]

\[ \int_{C_{k-1}}^{C_{k-1} + \mu} w'_k [x - C_{k-1}] \mathcal{L}_{k-1}(x) \, dx \right\} \]

\[ \Box \text{ Recall that} \]

\[ w_k(0^+) \triangleq w_k(c_k^{\min}) \]

and

\[ w'_k(\cdot) = 0 \quad \text{for} \quad (0, c_k^{\min}) \]
so that

\[ C_k \left( c_k^{\text{min}} \right) = p_k T \{ w_k \left( c_k^{\text{min}} \right) \mathcal{L}_{k-1} \left[ C_{k-1} \right] \} \]

Next, assume that we have a constant slope segment, i.e.,

\[ w'_k(x) = \lambda \quad x > c_k^{\text{min}} \]

and so

\[ C_k(x) = p_k T w_k \left( c_k^{\text{min}} \right) \mathcal{L}_{k-1} \left[ C_{k-1} \right] + \lambda_k \left[ \mathcal{E}_k(x) - \mathcal{E}_k \left( c_k^{\text{min}} \right) \right] \]
where

\[ \mathcal{E}_k(x) = p_k T \int_{C_{k-1}}^{C_{k-1} + x} \mathcal{L}_{k-1}(x) \, dx \]

In particular, for

\[ x = c_{k}^{\min} \]

we have

\[ \mathcal{C}_k\left(c_{k}^{\min}\right) = p_k T w_k\left(c_{k}^{\min}\right) \mathcal{L}_{k-1}\left[ C_{k-1} \right] \]
EVALUATION OF EXPECTED GENERATION COSTS

and we use this result to determine the expected generation costs for the first block loading of a multiple-block unit at the minimum capacity $c_k^{\min}$.

The loading of an additional block whose expected generation is $\xi_k$ leads to an associated expected value of the cost $c_k = \lambda_k \xi_k$. 
COMPUTATION OF EXPECTED EMISSIONS BY THERMAL UNITS

- A thermal unit produces emissions: the two principal ones of interest are
  - $CO_2$
  - $SO_2$

- A good estimate of the expected emission outputs is obtained by scaling the expected generation by a constant which is a unit specific characteristic of the effluent.
COMPUTATION OF EXPECTED EMISSIONS BY THERMAL UNITS

Once $\xi_k$ is computed for unit $k$, the emissions are evaluated through direct multiplication; for a particular technology we have specific parameters $\alpha_k^{CO_2}$ and $\alpha_k^{SO_2}$ to compute the expected effluents.

Then, compute the expected emissions from $\xi_k$

- $CO_2 \rightarrow \alpha_k^{CO_2} \xi_k$
- $SO_2 \rightarrow \alpha_k^{SO_2} \xi_k$
The quantity of emissions is an important attribute and needs to be evaluated to ensure compliance with environmental constraints.

A good application of the computation is to develop emission trading strategies such as those in $SO_2$ markets and the ones under current development in $CO_2$ markets.