ECE 588 – Electricity Resource Planning

2. Power System Reliability Concepts

George Gross
Department of Electrical and Computer Engineering
University of Illinois at Urbana-Champaign
Power system reliability has been and continues to be a primary concern in the management of electricity operations and planning.

Key drivers of the focus on reliability are:

- the utility obligation to serve load reliably and economically
POWER SYSTEM RELIABILITY

- the public’s expectation of a service of extremely high reliability: the light turns on whenever its switch is flipped

- the high costs incurred whenever outages occur because of the losses and damages to customers caused by food spoilage, production stoppage and loss of leisure
For a long time, the most common approach for making decisions on reliability was based on engineering judgment and experience with satisfactory results for power systems that are relatively small and simple.
As networks grew, empirical approaches were replaced by more rigorous methods based on formal quantitative/analytical techniques based on the application of probability and reliability theory; this statement, in general, is true for all systems having some degree of complexity.
The principal role is to provide predictions of the system’s behavior in the future based on past experiences.

Due to the inherent uncertainty in the process, predictions are by nature uncertain; while the truth of this statement is self-evident, some reliability studies to this day are not based on probability concepts [similarity exists here with most engineering design techniques that also involve predictions of future performance].

We adopt probabilistic thinking throughout the course.
DEFINITION OF RELIABILITY

- Reliability is the *probability* that a device, a component or a system performs its intended functions adequately for the specified time period and under the specified operating conditions.
- This definition conforms with the everyday concept of reliability.
- Reliability is measured in probability terms.
THE CONCEPT OF RELIABILITY

- The reliability of an electric supply system is defined as the probability of assurance of the provision of continuous service of satisfactory quality (with voltages and frequency within prescribed bounds) to the customers.

- For power systems, this definition of reliability is not used; instead this umbrella concept is replaced by two other notions.
THE CONCEPT OF RELIABILITY

- **Adequacy** is the ability of the system to supply the total energy and load demand of the customers within component ratings and voltage limits, with planned and unplanned component outages taken into account [system adequacy]

- **Security** is the ability of the system to withstand sudden disturbances such as the unanticipated losses of system components [system stability]
A third related concept is *system integrity* defined to be the ability to maintain *interconnected operations*, i.e., to avoid the separation of the system into islands whenever a severe disturbance occurs.

POWER SYSTEM OPERATIONAL TIME FRAME

- Transients
- Disturbance response
- Automatic system response
- Operator response
- Steady state operations
- Steady state contingencies

Operations horizon:
- $10^{-9}$ seconds
- $10^{-7}$ seconds
- $10^{-5}$ seconds
- $10^{-3}$ seconds
- $10^{-1}$ seconds
- 10 seconds

Planning horizon:
- Minutes
- Hours; days; months

Time scale:
- $10^{-9}$ seconds
- $10^{-7}$ seconds
- $10^{-5}$ seconds
- $10^{-3}$ seconds
- $10^{-1}$ seconds
- 10 seconds
- Minutes
- Hours; days; months
NATURE OF RELIABILITY STUDIES

- Adequacy evaluations
  - resources represented by probabilistic models
  - the network effects are, typically, represented deterministically: in effect, the assumption is that the network is not outaged and is able to deliver the energy injection at any node to any other node of the grid
- Planners can, typically, think in probabilistic terms unlike system operators
NATURE OF RELIABILITY STUDIES

- Operators do not think probabilistically since they consider principally the security aspects on a horizon extending a few hours or, at most, days and sometimes only minutes [fire-fighting mentality]

- The overriding concern of planners and operators is to make the system as reliable as is economically feasible
Common goal of both planners and operators is the maintenance of system adequacy, security and integrity at acceptable levels in order to avoid widespread outages.

Planners think about long-term goals with their objective being to ensure that the planned future system can adequately meet customers’ load and energy needs.
NATURE OF RELIABILITY STUDIES

- Long-term reliability studies in planning involve horizons as long as 10 – 30 years.
- Reliability studies involve the comparison of alternatives: the side-by-side evaluation of various alternatives to assess the reliability of each alternative.
- We require *relative measures* that are appropriate but are also more practical than *absolute measures* (issues of accuracy and computational burden).
NATURE OF RELIABILITY STUDIES

- Operator’s time horizons and associated goals:
  - short term of the order of hours [commitment/dispatch decisions]: provide adequate spinning reserves to minimize the occurrence of system failure through actions
    - to maintain the system ability to withstand sudden changes; and,
    - to choose appropriate corrective actions under the specified contingencies
SCOPE OF RELIABILITY STUDIES

- *longer horizons* of the order of weeks and months *resource scheduling decisions*: focus on scheduling issues, with, in certain cases, uncertainty represented in
  - maintenance scheduling
  - hydro scheduling

- *midterm periods* of the order of months up to 2 or 3 years *operational planning decisions*: undertake more planning-oriented studies such as mothballing of units or the return of units from retirement
THE PROBABILISTIC FRAMEWORK

- Probability theory deals with phenomena with uncertain outcomes that exhibit certain level of statistical regularity.

- We develop probabilistic models for
  - the resources; and,
  - the load.

- These models make use of basic concepts of probability theory.
EXAMPLE: COIN TOSSING

- We have a collection of outcomes which are discrete events; an event has the outcome of either $H$ or $T$.

- The probability of the outcome being $H$ is given by
EXAMPLE: COIN TOSSING

\[ P \{ \text{\( H \)} \} = \text{fraction of time the outcome is \( H \)} \]

for an even coin, \( P \{ \text{\( H \)} \} \rightarrow 50\% \)
EXAMPLE: COIN TOSSING

- We define the random variable (r.v.) $X$ to be

\[
X = \begin{cases} 
\text{H} & \text{if the outcome is H} \\
\text{T} & \text{otherwise}
\end{cases}
\]

- The probabilities $P\{\text{H}\}$ and $P\{\text{T}\}$ are determined from a frequency characterization: the fraction of times that the outcome is H and T, respectively.
CHARACTERIZATION OF THE CONTINUOUS \( r.v. \ X \)

\[ f_X(x) \]

\[ \text{probability density function} \]

\[ \text{probability} \]

\[ X \]
DISCRETE r.v. $X$ CHARACTERIZATION

$$f_X(x) = \sum_{i=1}^{n} p_i \delta(x - x_i)$$

probability

$p_1$ $p_2$ $\ldots$ $p_i$ $\ldots$ $p_{n-1}$ $p_n$

$x_1$ $x_2$ $\ldots$ $x_i$ $\ldots$ $x_{n-1}$ $x_n$
CHARACTERIZATION OF THE CONTINUOUS $r.v. \ X$

$$F_X(x) = P \{ X \leq x \}$$
DISCRETE $R.V. X$
CHARACTERIZATION

$$F_X(x) = \sum_{i=1}^{n} p_i u(x - x_i)$$

probability

1.0

$x_1$ $x_2$ $x_3$ ...

$x_n$
We model the load as the sum of individual demands; by summing over a large number of individual random demands, we obtain statistical regularity.

The regularity of the daily load profile is due to the

- work/rest pattern of demand
- seasonal nature of energy demand
- the underlying economy of the region
CHRONOLOGICAL LOAD MODEL

weekly summer load

MW

hours

Monday Tuesday Wednesday Thursday Friday Saturday Sunday
LOAD DURATION CURVE MODEL

weekly summer load

MW

0 38.71 168 hours

1 100 % fraction

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LOAD MODELING

- We think of the load $L$ as a r.v.: for each hour of the study period, the load has an outcome whose value is the hourly demand.

- We develop a probability distribution of $L$ from the hourly load curve for the given period: if we ignore time, we can rearrange the loads in order of decreasing values from the highest to the lowest and construct the load duration curve ($LDC$).
Load duration curve (LDC) at $t$ has the value $\ell$.

Interpretation of a point $(t, \ell)$ on the LDC: $t$ % of the time, the r.v. $\tilde{L} > \ell$. 
The base load $l_o$ is the minimum load for the period of study: the point where the curve has the value $l_o$ corresponds to 100% and its interpretation is that $L > l_o$ for 100% of the study period.

The peak load $l_p$ is the maximum load for the period of study; if the LDC has the value $l_p$ for a single point then it means that 0% of the time, the load r.v. $L > l_p$.
THE CHRONOLOGICAL LOAD CURVE

number of hours load value is greater than $\ell$

$$= t^A + t^B + t^C$$
GOING FROM THE CHRONOLOGICAL LOAD CURVE TO THE \textit{L.D.C.}

- For a given value \( \ell \), we compute

\[
\frac{\text{fraction of time above } \ell}{\ell} = \frac{\text{total number of hours in period load value is greater than } \ell}{\text{total number of hours in the period}}
\]

- For the example in the diagram

\[
\frac{\text{fraction of time above } \ell}{\ell} = \frac{t^A + t^B + t^C}{168}
\]
We then can evaluate

\[ F_L(\ell) = P \{ L \leq \ell \} = 1 - P \{ L > \ell \} = 1 - \mathcal{L}(\ell) \]

The curve \( \mathcal{L}(\cdot) \) is the complement of the c.d.f. of \( L \)

The use of the LDC implies the loss of all ideas of the chronological order of the loads in the period
UNIT AVAILABILITY

- A power system consists of many generation units of different ages, sizes and generation types
- Each unit is a complex system in itself with many possible modes of operation and modes of failure
- The simplest representation of unit availability is a two-state model with
UNIT AVAILABILITY

- Full unit on outage with capacity \( c \)
- Full operating capacity \( c \)
- Full unit on outage with capacity \( 0 \)

\( up \)
\( down \)
\( time \)
TWO-STATE MODEL OF UNIT AVAILABILITY

- We have two possible states for the unit
  - unit is *up* with full capacity $c$
  - unit is *down* with capacity 0

- We define the r.v. $\tilde{A}$ to denote the unit available capacity

\[
\tilde{A} = \begin{cases} 
\text{capacity of unit} & \text{if unit is up} \\ 
0 & \text{if unit is on outage} 
\end{cases}
\]

- We write $P\{\text{unit down}\}$ as shorthand for $P\{\tilde{A} = 0\}$
  
  and $P\{\text{unit up}\}$ for $P\{\tilde{A} = \text{capacity of unit}\}$
TWO-STATE MODEL OF UNIT AVAILABILITY

- Since $A$ can have only two outcomes
  
  \[ P\{\text{unit up}\} + P\{\text{unit down}\} = 1 \]

- We compute
  
  \[ P\{\text{unit up}\} = \frac{\text{number of hours unit up}}{\text{number of hours unit up} + \text{number of hours unit down}} \]

- We define
  
  \[ P\{\text{unit up}\} \] to be the availability of the unit

  \[ P\{\text{unit down}\} \] to be the unavailability of the unit
TWO-STATE MODEL OF UNIT AVAILABILITY

- We call *service hours* the total number of hours unit is electrically connected to serve load; this number excludes the scheduled maintenance times.

- The number of forced outage hours (*FOH*) is the number of hours unit is unavailable because of an unplanned component failure – such as, startup immediate outage or a delayed outage – and requires that the unit be removed from service immediately.
TWO-STATE MODEL OF UNIT AVAILABILITY

- We use the following defining relationship for forced outage rate or $FOR$

$$FOR = \frac{FOH}{service\ hours} \cdot 100\%$$

- The $FOR$ terminology is a misnomer since it is not a rate but simply a ratio
TWO – STATE MODEL OF UNIT AVAILABILITY

\[ A_i = \begin{cases} 
  c_i & \text{unit } i \text{ is available with probability } p_i \\
  0 & \text{unit } i \text{ is forced out with probability } (1 - p_i) 
\end{cases} \]

\[ f_{A_i}(x) = (1 - p_i) \delta(x) + p_i \delta(x - c_i) \]

\[ F_{A_i}(x) = (1 - p_i) u(x) + p_i u(x - c_i) \]
CONVOLUTION

- We consider two independent r.v.s \( X \) and \( Y \).

- We wish to compute the distribution of

\[
\tilde{Z} = \tilde{X} + \tilde{Y}
\]

- The computation uses the convolution operation:

\[
F_{\tilde{X}+\tilde{Y}}(z) = P\{ \tilde{X} + \tilde{Y} \leq z \} = P\{ \tilde{X} \leq z - \tilde{Y} \}
\]
CONVOLUTION

by the law of total probability

\[
= \int_{-\infty}^{\infty} P \left\{ X \leq z - y \right\} f_Y(y) dy
\]

use of the statistical independence assumption

\[
= \int_{-\infty}^{\infty} P \left\{ X \leq z - y \right\} F_X(z - y) dy
\]

\[
= \int_{-\infty}^{\infty} F_X(z - y) f_Y(y) dy
\]

\[
= \int_{-\infty}^{\infty} F_Y(z - x) f_X(x) dx
\]
CONVOLUTION : SIMPLE APPLICATION

- We compute the sum of availabilities of two units using convolution

\[ X = \mathcal{A}_1 \quad \text{and} \quad Y = \mathcal{A}_2 \]

- Convolution obtains

\[ P \left\{ \mathcal{A}_1 + \mathcal{A}_2 \leq x \right\} = P \left\{ \mathcal{A}_1 \leq x \right\} (1 - p_2) + P \left\{ \mathcal{A}_1 \leq x - c_2 \right\} p_2 \]

\[ = \left( F_{\mathcal{A}_1}(x) \right) (1 - p_2) + \left( F_{\mathcal{A}_1}(x - c_2) \right) p_2 \]

\[ \text{c.d.f. of } \mathcal{A}_1 \quad \text{c.d.f. of } \mathcal{A}_1 \text{ delayed by } c_2 \]
THE STATE SPACE OF THE SIMPLE APPLICATION

<table>
<thead>
<tr>
<th>r.v. realization</th>
<th>realization probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{A}_1$</td>
<td>$\tilde{A}_2$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$c_1$</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>$c_2$</td>
</tr>
<tr>
<td>$c_1$</td>
<td>$c_2$</td>
</tr>
</tbody>
</table>
THE PROBABILITY DENSITY OF $A_1 + A_2$

\[ p_1(1 - p_2)(1 - p_1)p_2 \]

\[ (1 - p_1)(1 - p_2) \]

\[ 0 \]

\[ c_1 \]

\[ c_2 \]

\[ c_1 + c_2 \]

\[ MW \]

\[ p_1p_2 \]

\[ \text{probability} \]
THE CUMULATIVE DISTRIBUTION FUNCTION OF $\tilde{A}_1 + \tilde{A}_2$

probability

$\frac{(1 - p_1)(1 - p_2)}{1 - p_1 p_2}$

$\frac{1 - p_2}{1 - p_1 p_2}$

$\frac{1 - p_1 p_2}{MW}$

$O \quad c_1 \quad c_2 \quad c_1 + c_2$
Let

\[ X_\sim = \sum_{j=1}^{i-1} A_j \]

and

\[ Y_\sim = A_i \]

We compute the c.d.f. of \( \sum_{j=1}^{i} A_j \) using

\[
P \{ X_\sim + Y_\sim \leq z \} = (1 - p_i) P \{ X_\sim \leq z \} + p_i P \{ X_\sim \leq z - c_i \}
\]

\[
= (1 - p_i) P \left\{ \sum_{j=1}^{i-1} A_j \leq z \right\} + p_i P \left\{ \sum_{j=1}^{i-1} A_j \leq z - c_i \right\}
\]
# EXAMPLE: THREE-UNIT SYSTEM

<table>
<thead>
<tr>
<th>unit i</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_i$</td>
<td>400</td>
<td>500</td>
<td>600</td>
</tr>
<tr>
<td>$p_i$</td>
<td>0.99</td>
<td>0.98</td>
<td>0.97</td>
</tr>
<tr>
<td>$1 - p_i$</td>
<td>0.01</td>
<td>0.02</td>
<td>0.03</td>
</tr>
</tbody>
</table>
**EXAMPLE: THREE-UNIT SYSTEM**

<table>
<thead>
<tr>
<th>unit(s)</th>
<th>capacity states and associated probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1 only</strong></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>400</td>
</tr>
<tr>
<td>.01</td>
<td>.99</td>
</tr>
<tr>
<td><strong>1 and 2</strong></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>400</td>
</tr>
<tr>
<td>.0002</td>
<td>.99 x .02</td>
</tr>
<tr>
<td></td>
<td>.0198</td>
</tr>
<tr>
<td>.98 x .01</td>
<td>.0098</td>
</tr>
<tr>
<td>.99 x .98</td>
<td>.9799</td>
</tr>
<tr>
<td><strong>1 and 2 and 3</strong></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>400</td>
</tr>
<tr>
<td>.000006</td>
<td>0.000594</td>
</tr>
<tr>
<td>0.000294</td>
<td>0.000194</td>
</tr>
<tr>
<td>0.029106</td>
<td>0.019206</td>
</tr>
<tr>
<td>0.009506</td>
<td>0.941094</td>
</tr>
<tr>
<td>900</td>
<td>1000</td>
</tr>
<tr>
<td>1100</td>
<td>1500</td>
</tr>
</tbody>
</table>
RELIABILITY INDICES

- $\text{LOLP}$: loss of load probability
- $\text{LOLE}$: loss of load expectation
- $\text{EUE}$: expected unserved energy
LOLP

- We define \( LOLP \) to be the probability of the event that the load exceeds the available generation capacity.

- Mathematically, we use the relationship

\[
LOLP \triangleq P \left\{ \sum_i A_i < L \right\}
\]

- In actual practice, the computation of the values of this index is implemented making use of an availability table or an outage table:
LOLP

- availability table: values of the possible generation capacities and associated probabilities

- capacity table: values of the possible outage capacities and associated probabilities
LOLP

- The outage capacity is defined by

\[ Z_i \triangleq c_i - A_i \]

- The determination of the LOLP index requires its evaluation for possible load values \( \ell \):

\[ LOLP(\ell) = P \left\{ \sum_i A_i < \ell \right\} \]

- We take care to distinguish between the function \( LOLP(\bullet) \) and its values at different load levels.
$$LOLP(\ell)$$

\[
LOLP(\ell) = P \left\{ \sum_i A_i < \ell \right\}
\]

\[
= P \left\{ \sum_i (c_i - Z_i) < \ell \right\}
\]

\[
= P \left\{ \sum_i c_i - \ell < \sum_i Z_i \right\}
\]

\[
= P \left\{ \sum_i Z_i > \sum_i c_i - \ell \right\}
\]

(outage capacity)

(installed capacity)

(demand level)
EXAMPLE: THREE-UNIT SYSTEM WITH $\ell = 600 \, MW$

$$LOLP(600) = P\{A_1 + A_2 + A_3 < 600\}$$

$$LOLP(600) = .000894$$

obtained from row 3 of the table by summing up the entries for the capacity states 0, 400, and 500 MW
EXAMPLE: THREE-UNIT SYSTEM WITH $\ell = 800 \text{ MW}$

$$LOLP(800) = P \left\{ \tilde{A}_1 + \tilde{A}_2 + \tilde{A}_3 < 800 \right\}$$

$$LOLP(800) = 0.001088$$

obtained from row 3 of the table by summing up the entries for the capacity states 0, 400, 500 and 600 MW
EXAMPLE: THREE–UNIT SYSTEM WITH $\ell = 1100 \text{ MW}$

$$LOLP(1100) = P\{A_1 + A_2 + A_3 < 1100\}$$

$$LOLP(1100) = 0.049400$$

obtained from row 3 of the table by summing up the entries for the capacity states 0, 400, 500, 600, 900, and 1000
EXAMPLE: THREE-UNIT SYSTEM WITH $\ell = 1200 \, MW$

\[
LOLP (1200) = P \left\{ A_1 + A_2 + A_3 < 1200 \right\}
\]

\[
LOLP (1200) = .058906
\]

obtained from row 3 of the table

by summing up the entries for the capacity states $0, 400, 500, 600, 900, 1000,$ and $1100 \, MW$
HOURLY *LOLP* CALCULATIONS

- Let the load in hour $h$ be $\ell$; then, $LOLP(\ell)$ corresponds to the hour $h$ LOLP value and may be denoted as $LOLP_h$.

- The interpretation of $LOLP_h$ is the probability that there is a loss of load in the hour $h$.

- The evaluation of $LOLP_{\mathcal{I}}$ for a specified time period $\mathcal{I}$, where the period consists of the hours $\mathcal{I} = \{h_1, h_2, \ldots, h_T\}$, is computed using conditional probability.
HOU RYL \textit{LOLP} CALCULATIONS

\[ \text{LOLP}_\mathcal{T} = P\{\text{loss of load in period } \mathcal{T}\} \]

\[ = P\{\text{loss of load in any hour of period } \mathcal{T}\} \]

\[ = P\{\text{loss of load in hour } h_1, h_2, \ldots, h_T\} \]

\[ = P\{\text{loss of load in hour } h_1\} \cdot P\{\text{hour } h_1\} + \]

\[ \ldots + P\{\text{loss of load in hour } h_T\} \cdot P\{\text{hour } h_T\} \]
HOURLY \textit{LOLP} CALCULATIONS

\[ \text{LOLP}_{\mathcal{J}} = \sum_{i=1}^{T} \text{LOLP}_{h_i} \cdot \frac{1}{T} \]

- The interpretation of \( \text{LOLP}_{\mathcal{J}} \) is the probability that a loss of load occurs in any hour in the set \( \{h_1, h_2, \ldots, h_T\} \) of the specified period \( \mathcal{J} \).
DAILY *LOLP* CALCULATIONS

- Consider the situation with the smallest time granularity of a day and let $\ell$ be the load for the day $d$
- Typically, $\ell$ represents the peak load of day $d$; note that this is different from the case when an hourly time granularity is used since under the latter case, for each hour of the day, a different load level may exist
DAILY LOLP CALCULATIONS

\[ \ell \]

\[ \ell \]

hour \( h \)

day \( d \)
DAILY LOLP CALCULATIONS

- \( LOLP(\ell) \) in this case corresponds to the day \( d \)

\( LOLP \) value and may be denoted by \( LOLP_d \)

- In a fashion analogous to the hourly \( LOLP \) calculations we can show that for, say, a week period

\[
LOLP_{\text{week}} = \frac{1}{7} \sum_{i=1}^{7} LOLP_{d_i}
\]
EXAMPLE: THREE-UNIT SYSTEM
WEEKLY \textit{LOLP} CALCULATION

<table>
<thead>
<tr>
<th>day</th>
<th>Su</th>
<th>Mo</th>
<th>Tu</th>
<th>We</th>
<th>Th</th>
<th>Fr</th>
<th>Sa</th>
</tr>
</thead>
<tbody>
<tr>
<td>load</td>
<td>600</td>
<td>1100</td>
<td>1200</td>
<td>1100</td>
<td>1200</td>
<td>1200</td>
<td>800</td>
</tr>
</tbody>
</table>
EXAMPLE: THREE-UNIT SYSTEM WEEKLY LOLP CALCULATION

Conditional probability calculation

\[ \text{LOLP}_{\text{week}} = \sum_{i=1}^{7} \text{LOLP}_{d_i} P\{\text{day} = d_i\} \]

\[ = \frac{1}{7} \sum_{i=1}^{7} \text{LOLP}_{d_i} \]

\[ = \left(0.2775\right) \frac{1}{7} \cdot \frac{7 \text{ days}}{\text{week}} \]

\[ = 0.2775 \text{ days / week} \]
EXAMPLE: THREE-UNIT SYSTEM WEEKLY $LOLP$ CALCULATION

- The meaning of

\[ LOLP_{\text{week}} = 0.2775 \text{ days / week} \]

is that the probability of loss of load is

1 day in 3.6036 weeks
LOLE

- The meaning of LOLE is the expected value of the number of loss of load (l.o.l.) events for the given time resolution and over the specified period.
- LOLE has a frequency interpretation and is measured in number of events/duration of the specified period.
- The LOLE index is typically defined for both daily and hourly values.
LOLE

- For the daily LOLE, the number of events is evaluated in terms of days out of the total number of days in the period.
- For the hourly LOLE, the number of events is evaluated in terms of hours out of the total number of hours in the period.
- We define a binary valued r.v. $N_i \sim$ for time unit $i$:
  $$N_i \triangleq \begin{cases} 
 1 & \text{if l.o.l. occurs in unit of time } i \\
 0 & \text{otherwise}
  \end{cases}$$
If $\ell_i$ is the load during time unit $i$, the probabilities of $N_i$ are given by

$$
P\{N_i = k\} = \begin{cases} 
\text{LOLP}(\ell_i) & \text{if } k = 1 \\
1 - \text{LOLP}(\ell_i) & \text{if } k = 0 
\end{cases}
$$
DAILY \textit{LOLE}

- For the daily \textit{LOLE}

\[ N_i \triangleq \begin{cases} 
1 & \text{if l.o.l. on day } d_i \\
0 & \text{otherwise}
\end{cases} \]

- Then for the specified period of \( D \) days, the value

\[ \text{LOLE}_d \triangleq E \left\{ \sum_{i=1}^{D} N_i \right\} \frac{\text{days}}{D \text{ days}} \] expected number of days out of \( D \) days with l.o.l.
DAILY \textit{LOLE}

\[ \frac{D}{days} = \sum_{i=1}^{D} E \left\{ N_{d_i} \right\} \frac{days}{D \text{ days}} \]

\[ = \sum_{i=1}^{D} \left[ 1 \cdot \text{LOLP} \left( \ell_i \right) + \theta \cdot \left( 1 - \text{LOLP} \left( \ell_i \right) \right) \right] \frac{days}{D \text{ days}} \]

\[ = \sum_{i=1}^{D} \text{LOLP} \left( \ell_i \right) \frac{days}{D \text{ days}} \]

- For a period of one \textit{year}, \textit{i.e.,} $D = 365 \text{ days}$, the value is given by
DAILY LOLE

\[ \text{LOLE}_d = \sum_{i=1}^{365} \text{LOLP}(\ell_i) \frac{\text{days}}{365 \text{ days}} \cdot \frac{365 \text{ days}}{\text{year}} \]

\[ = \sum_{i=1}^{365} \text{LOLP}(\ell_i) \frac{\text{days}}{\text{year}} \]

- The daily LOLE is typically expressed in 1 day in \( xx \) years

- So,

\[ xx = \frac{1}{\text{LOLE}_d} \text{ years} \]
HOURLY LOLE

- In hourly LOLE computation, the smallest time unit is one hour.

- We define for this case the binary valued r.v.

\[ N_i \triangleq \begin{cases} 
1 & \text{l.o.l. in hour } i \\
0 & \text{otherwise} 
\end{cases} \]

- For a period of \( H \) hours we compute the value...
HOURLY \( LOLE \)

\[
LOLE_h \triangleq E \left\{ \sum_{i=1}^{H} N_i \right\} \frac{\text{hours}}{H \text{ hours}}
\]

\[
= \sum_{i=1}^{H} LOLP(\ell_i) \frac{\text{hours}}{H \text{ hours}}
\]

- It is customary to express \( LOLE_h \) in units of \( \text{hours/year} \) and so we bring in the conversion factor of \( 1 \text{ year} = 8760 \text{ hours} \)

- For resource capacity assessments, the \( NPCC \) \( LOLE \) criterion index is \( LOLE \leq 0.1 \text{ day/year} \)
HOURLY LOLE

\[
\text{LOLE}_h = \sum_{i=1}^{H} \text{LOLP}(\ell_i) \frac{\text{hours}}{H \text{ hours}} \cdot \frac{8760 \text{ hours}}{\text{year}} \\
= \sum_{i=1}^{H} \text{LOLP}(\ell_i) \frac{8760}{H} \frac{\text{hours}}{\text{year}}
\]

- Sometimes, it is desired to express \( \text{LOLE}_h \) in units of 1 hour in \( xx \) years and for an \( H \)-hour period

\[
xx = \frac{1}{\text{LOLE}_h} \text{ years} = \frac{H / 8760}{\sum_{i=1}^{H} \text{LOLP}(\ell_i)} \text{ years}
\]
LOSS OF LOAD HOURS \((LOLH)\)

- We define \(LOLH\) expressed in \(\text{hours/year}\) to be the \(LOLE\) index with the use of hourly resolution for the load representation

\[
LOLH = LOLE_h
\]

- Hence, the evaluation of \(LOLH\) for a 1–week period requires the computation of the 168 \(LOLP\) values; in contrast, the \(LOLE_d\) evaluation needs only 7 \(LOLP\) values, one for each daily peak
HOURLY LOAD MODEL FOR LOLH EVALUATION

hourly load values (MW)

1  24  48  72  96  120  144  168  hour

0  2000  4000  6000  8000  10000  12000  14000  16000
DAILY LOAD MODEL FOR $LOLE_d$ EVALUATION

Daily load values (MW)

<table>
<thead>
<tr>
<th>Day</th>
<th>Load Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13,927</td>
</tr>
<tr>
<td>2</td>
<td>14,000</td>
</tr>
<tr>
<td>3</td>
<td>14,324</td>
</tr>
<tr>
<td>4</td>
<td>14,166</td>
</tr>
<tr>
<td>5</td>
<td>13,659</td>
</tr>
<tr>
<td>6</td>
<td>13,029</td>
</tr>
<tr>
<td>7</td>
<td>12,982</td>
</tr>
</tbody>
</table>
FREQUENT MISINTERPRETATIONS

- The $LOLE_d$ index captures neither the duration nor the magnitude of the l.o.l. event and so

$$1 \text{ day in 10 years } LOLE_d \neq 24 \text{ hours in 10 years } LOLH$$

- As an example, consider the case in which 2 hours of firm load shed occur in a single day of a given year; this l.o.l. event results in $LOLH = 2 \text{ hours/year}$, but $LOLE_d = 1 \text{ day/year}$

- No direct translations are possible from $LOLH$ to $LOLE_d$ and vice versa without the evaluation of each index with the respective load model
SIMULATION STUDIES: NYISO

- study system: New York state control area
- simulation year: 2016
- system peak load: 33,928 MW
- total installed capacity: 39,330 MW
- load model: 8760 hourly values for the given study period
- objective: evaluation of \( \text{LOLH} \) when under the 0.1 \( \text{LOLE}_d \) criterion and of the \( \text{LOLE}_d \) under the 2.4 \( \text{LOLH} \) criterion
**CASE STUDY RESULTS**

<table>
<thead>
<tr>
<th>specified criterion</th>
<th>evaluated value</th>
</tr>
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<tbody>
<tr>
<td>$LOLE_d = 0.1$ day/year</td>
<td>$LOLH = 0.31$ hour/year</td>
</tr>
<tr>
<td>$LOLH = 2.4$ hours/year</td>
<td>$LOLE_d = 0.63$ day/year</td>
</tr>
</tbody>
</table>

*6 times higher than the $LOLE_d$ criterion value*
LOLE_d AND LOLE_h

- A limit-type relationship may be derived between the two LOLE indices
- Under the assumption that the daily load used in the evaluation of the LOLE_d index is computed based on the peak hourly load, we can show that

\[ \text{LOLE}_h \leq 24 \text{LOLE}_d \]

for every time period
The $EUE$ is a very important reliability metric since it provides a more meaningful measure of the impact of loss of load.

We use $\mathcal{U}$ to denote $EUE$ with the definition that $\mathcal{U}$ is the expected value of energy not served due to inadequate available capacity.

Mathematically, we have for any period $\mathcal{T}$
$\mathcal{U} = EUE$

\[
= \sum_{h \in \mathcal{F}} E \left\{ MWH \text{ not served in hour } h \right\}
\]

\[
= \sum_{h \in \mathcal{F}} E \left\{ MWH \text{ not served in hour } h \right\}
\]

\[
= \sum_{h \in \mathcal{F}} E \left\{ MWH \text{ not served } \mid \text{ hour } h \right\} \cdot P \left\{ \text{hour } h \right\}
\]

\[
\max \left\{ 0, \ell_h - \sum_{i=1}^{N} A_i \right\} \quad \text{uniformly distributed with probability} \quad \frac{1}{T}
\]
We define the capacity deficiency r.v. for hour $h$

$$D_h \triangleq \begin{cases} 
\ell_h - \sum_{i=1}^{N} A_i & \text{if } L > \sum_{i=1}^{N} A_i \\
0 & \text{otherwise}
\end{cases}$$

We evaluate

$$\mathcal{U} = \frac{1}{T} \sum_{h=1}^{T} E\{D_h\}$$
The units of $U$ are $MWh/h$ for the period $T$ of $T$ hours; for a period $T$ consisting of the 8,760 hours in a year then $U$ reduces to

$$U^{year} = \sum_{h=1}^{8,760} E \{ D_h \} MWh / \text{year}$$

As closely related index is $EDNS$ which measures the expected demand not served and is expressed in $MW$
EXAMPLE: THREE-UNIT SYSTEM

- We are interested in computing the value of $\mathcal{U}$ for a single hour $T = 1$ with a load of 600 MW.
- We compute

$$\mathcal{U}(600) = \left(\frac{600 - 0}{600}\right) \cdot P\{0\} + \left(\frac{600 - 400}{200}\right) \cdot P\{400\}$$

$$+ \left(\frac{600 - 500}{100}\right) \cdot P\{500\}$$

$$= \frac{600}{600} \cdot 0.000006 + \frac{200}{200} \cdot 0.000594$$

$$+ \frac{100}{100} \cdot 0.000294$$

$$= 0.1518 \text{ MWh}$$
SIMULATION STUDIES: NYISO

- We repeat the same simulations with the goal to evaluate the EUE.
- We define the normalized EUE:
  \[ \mathcal{U}_n = \frac{\mathcal{U}}{E} \], \( E \) denotes the total energy produced in the system.
- We impose a reliability criterion of \( \mathcal{U}_n = 0.002 \) or equivalently \( \mathcal{U} = 320 \ MWh/year \) encountered in the Australia’s National Energy Market and recalculate \( \text{LOLE}_d \) and \( \text{LOLH} \).
### CASE STUDY RESULTS

<table>
<thead>
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<td>( \text{LOLE}_d = 0.1 \text{ day/year} )</td>
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</table>
BASIC ASSUMPTIONS IN RELIABILITY MODELING

- The only components of the system considered are the generation units and the load.
- The T&D system provides the linkage between the generation units and the load is assumed to be 100% available and to have no constraints on transfer capability.
- Statistical independence of the r.v.s.
  - \( \overline{A}_i \) independent of \( \overline{A}_j \), \( i \neq j \), \( i, j = 1, 2, \ldots, N \)
  - each \( \overline{A}_i \), \( i = 1, 2, \ldots, N \) is independent of the load r.v. \( \overline{L} \)
BASIC MODEL

- The availability of each unit $i$ with capacity $c_i$ is represented by r.v. $\tilde{A}_i$ with

\[
\tilde{A}_i = \begin{cases} 
  c_i \text{ unit is available with probability } p_i \\
  0 \text{ unit is unavailable with probability } 1 - p_i 
\end{cases}
\]

- The load r.v. $\tilde{L}$ has a distribution obtained from the chronological load or the LDC with the complement of the c.d.f. given by

\[
\mathcal{L}(\ell) = P \{ \tilde{L} > \ell \}
\]
BASIC CONVOLUTION FORMULA

\[
P \left\{ \sum_{i=1}^{n} A_i \leq x \right\} = P \left\{ \sum_{i=1}^{n-1} A_i + A_n \leq x \right\}
\]

\[
= P \left\{ \sum_{i=1}^{n-1} A_i + c_n \leq x \, \big| \, A_n = c_n \right\} P \left\{ A_n = c_n \right\} + \]

\[
P \left\{ \sum_{i=1}^{n-1} A_i \leq x \, \big| \, A_n = 0 \right\} P \left\{ A_n = 0 \right\}
\]

\[
P \left\{ \sum_{i=1}^{n} A_i \leq x \right\} = (1 - p_n) P \left\{ \sum_{i=1}^{n-1} A_i \leq x \right\} + p_n P \left\{ \sum_{i=1}^{n-1} A_i \leq x - c_n \right\}
\]
LOLP EVALUATION

- Recall that the LOLP is computed using

\[ \text{LOLP} = P \left\{ \sum_{i=1}^{N} A_i < L \right\} \]

- Consider the load r.v. \( \sim L \) with a discrete distribution with discrete values \( \{ \ell_j, j = 1, 2, ..., n_\ell \} \) so that

\[ \sum_{j=1}^{n_\ell} P \left\{ L = \ell_j \right\} = 1 \]
EVALUATION OF \( \text{LOLP} \)

Therefore,

\[
\text{LOLP} = \sum_{j=1}^{n_t} P \left\{ \sum_{i=1}^{N} A_i < \ell_j \mid L_j = \ell_j \right\} P \left\{ L_j = \ell_j \right\}
\]

and we write

\[
\text{LOLP} = \sum_{j=1}^{n_t} \text{LOLP}(\ell_j) \cdot P \left\{ L_j = \ell_j \right\}
\]
EVALUATION OF $\mathcal{U}$

- We define the capacity deficiency r.v.

$$D \triangleq \begin{cases} 
L - \sum_{i=1}^{N} A_{i} & \text{for } L > \sum_{i=1}^{N} A_{i} \\
0 & \text{otherwise}
\end{cases}$$

- We evaluate

$$\mathcal{U} = E\{D\} = E\left\{L - \sum_{i=1}^{N} A_{i} \middle| \sum_{i=1}^{N} A_{i} \right\} P\left\{L > \sum_{i=1}^{N} A_{i} \right\} + 0$$
EVALUATION OF $\mathcal{U}$

- For a discrete distribution r.v. $L$

$$\mathcal{U} = \sum_{j=1}^{n_L} E \left\{ \ell_j - \sum_{i=1}^{N} A_i \bigg| \ell_j > \sum_{i=1}^{N} A_i \right\} \cdot LOLP(\ell_j) \cdot P \left\{ L = \ell_j \right\}$$