Problem 1

Consider

\[(w_j - \frac{1}{\eta_j \eta_c} k_j \rho - \frac{1}{\eta_c} v_j) g_j = 0 \quad j = 1, \ldots, J\]

Line \( v_j > 0 \) if unit \( j \) is used to charge.

\[w_j = \{(\rho - \beta) k_j - \beta\} \eta_j \eta_c = 0 \quad j = 1, \ldots, K\]

\[w_j = \{(\rho - \beta) k_j - \beta\} \eta_j \eta_c\]

We know that \( \beta > 0 \) and \( v_j > 0 \)

\[w_j = \rho k_j \eta_j \eta_c \quad j = 1, \ldots, K\]

Let \( \lambda_c = \max \left\{ \frac{w_j}{k_j} : j = 1, \ldots, K \right\} \)

Then,

\[\frac{w_j}{k_j} \leq \rho \eta_j \eta_c \quad \forall j = 1, \ldots, K \Rightarrow \lambda_c \leq \rho \eta_j \eta_c\]

Consider

\[\left[w_j - \{(\rho - \beta) k_j - \beta\} \eta_j \eta_c\right] v_j = 0 \quad j = 1, \ldots, K\]

Line \( g_j > 0 \) if storage generates

\[w_j = \frac{1}{\eta_j \eta_c} k_j \rho - \frac{1}{\eta_c} v_j = 0 \quad j = 1, \ldots, J\]

or

\[w_j = \frac{1}{\eta_j \eta_c} k_j \rho + \frac{1}{\eta_c} v_j\]

Now \( v_j > 0 \) \( \Rightarrow \)

\[w_j \geq \frac{1}{\eta_j \eta_c} k_j \rho\]

Let \( \lambda_g = \min \left\{ \frac{w_j}{k_j} : j = 1, \ldots, J \right\} \)

Then,

\[\frac{w_j}{k_j} > \frac{1}{\eta_j \eta_c} / \forall j = 1, \ldots, J \Rightarrow \lambda_g > \frac{1}{\eta_j \eta_c} \rho \text{ QED} \]
Problem 2

\[ E_j^e = \text{EXPECTED ENERGY THAT CAN BE USED FOR CHARGING AN ENERGY STORAGE PLANT PROVIDED BY UNIT } j \]

(i) \( E_j^e = ? \) if \( j \) is the first block of a 3-state unit.

Generally, the energy available to charge a storage plant with unit \( j \) is determined according to the shaded area as indicated below:

\[ E_j^e = p_j T \left( c_j - \int_{c_{j-1}}^{c_j} \chi_{j-1}(x) \, dx \right) \]

\[ = p_j T c_j - E_j \]

If \( j \) is the first block of a 3-state unit:

\[ E_j^e = T \left\{ p_j \int_{c_{j-1}}^{c_j} (1 - \chi_{j-1}(x)) \, dx + s_j \int_{c_{j-1}}^{c_j} (1 - \chi_{j-1}(x)) \, dx + d_j \right\} - E_j \]

\[ = T \left\{ p_j d_j + s_j (c_j - d_j) \right\} - E_j \]

\[ = T \left\{ d_j (p_j - s_j) + s_j c_j \right\} - E_j \]

\[ = (r_j d_j + s_j c_j) - E_j \quad \text{for } c_j > d_j \]

If \( c_j < d_j \)

Then it practically becomes a two-state unit.
(ii) If \( j \) is the second block of a 2-state unit.

Assuming that \( i \) is the first block to be loaded and the total capacity of the 2-state unit is \( q_i \):

The probability of the unit being available is \( p_j \).

The energy available from \( j \) for charging the storage unit is equivalent to the incremental case previously described.

When the second block is charged, the available energy for charging is determined according to the shaded area:

\[
\tilde{E}_j = p_j \int_{q_{j-1}}^{q_j} (1 - F_j(x)) \, dx
\]

where \( F_j(x) = 1 - F_2(x) \)

where \( \tilde{I}_j \) is the equivalent load resulting from loading all units except \( i \).
Problem 3

We have the set $\mathcal{S}$ of $SP$’s

$$\mathcal{S} = \{s_1, s_2, \ldots, s_m\}$$

with $\eta_i \geq \eta_{i+1} \quad \forall i = 1, 2, \ldots (m - 1)$.

We adapt the algorithm in order to be able to simulate all of them: