• Problem 1: Every multi-state unit has the following distribution function:

\[ f_{A_i}(x) = p_i \delta(x - c_i) + \sum_{j=1}^{k-2} s_i^j \delta(x - d_i^j) + (1 - \sum_{j=1}^{k-2} s_i^j - p_i) \delta(x) \]  

(1)

Where 0 < \(d_i^{k-2} < d_i^{k-3} < \ldots d_i^1 < c_i\).

To find the distribution function of the sum of a set of this random variables, due to statistical independence, we can use the fact that the distribution of \(A = A_1 + A_2 + \ldots + A_n\) is equal to the n-convolution of the distributions,

\[ f_A(x) = f_{A_1}(x) * f_{A_2}(x) * f_{A_3}(x) * \ldots * f_{A_n}(x) \]  

(2)

The above equation is the generalization to the case with 2 random variables like lecture notes. Then using this distribution, we can find \(P\{i \leq x\}\) in the following way,

\[ P\{i \leq x\} = \int_0^x f_A(y)dy \]  

(3)

We can also find an equivalent formula following the steps in lecture notes for the case of two-state system.

Let

\[ Z = \sum_{j=1}^{i-1} A_j \]  

(4)

and

\[ Y = A_i \]  

(5)

, then

\[ P\{\sum_{l=1}^i A_l \leq x\} = F_Z + Y(x) = \int_{-\infty}^\infty F_Z(x - m)f_Y(m)dm \]  

(6)

Using (1) into (6) we obtain that,

\[ P\{\sum_{l=1}^i A_l \leq x\} = (1 - \sum_{j=1}^{k-2} s_i^j - p_i)P\{Z \leq x\} + \sum_{j=1}^{k-2} s_i^j P\{Z \leq x - d_i^j\} + p_i P\{Z \leq x - c_i\} \]  

(7)
• Problem 2 :

<table>
<thead>
<tr>
<th>i</th>
<th>$c_i$(MW)</th>
<th>$p_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>400</td>
<td>.99</td>
</tr>
<tr>
<td>2</td>
<td>500</td>
<td>.98</td>
</tr>
<tr>
<td>3</td>
<td>600</td>
<td>.97</td>
</tr>
</tbody>
</table>

From the above table it is possible to build first a table from operation of units 1 and 2:

- Capacity=0, 1 down and 2 down, $(1 - p_1)(1 - p_2) = 0.01 \times 0.02 = 0.0002$
- Capacity=400, 1 up and 2 down, $(p_1)(1 - p_2) = 0.99 \times 0.02 = 0.0198$
- Capacity=500, 1 down and 2 up, $(1 - p_1)(p_2) = 0.01 \times 0.98 = 0.0098$
- Capacity=900, 1 up and 2 up, $(p_1)(p_2) = 0.99 \times 0.98 = 0.9799 = 0.9702$

Now adding unit 3, we obtain:

- Capacity=0, 1 down, 2 down and 3 down, $(1 - p_1)(1 - p_2)(1 - p_3) = 0.0002 \times 0.03 = 0.000006$
- Capacity=400, 1 up, 2 down and 3 down, $(p_1)(1 - p_2)(1 - p_3) = 0.0198 \times 0.03 = 0.000594$
- Capacity=500, 1 down, 2 up and 3 down, $(1 - p_1)(p_2)(1 - p_3) = 0.0098 \times 0.03 = 0.002994$
- Capacity=600, 1 down, 2 down and 3 up, $(1 - p_1)(1 - p_2)(p_3) = 0.0002 \times 0.97 = 0.0000194$
- Capacity=900, 1 up, 2 up and 3 down, $(p_1)(p_2)(1 - p_3) = 0.9702 \times 0.03 = 0.029106$
- Capacity=1000, 1 up, 2 down and 3 up, $(p_1)(1 - p_2)(p_3) = 0.0198 \times 0.97 = 0.019206$
- Capacity=1100, 1 down, 2 up and 3 up, $(1 - p_1)(p_2)(p_3) = 0.0098 \times 0.97 = 0.009506$
- Capacity=1500, 1 up, 2 up and 3 up, $(p_1)(p_2)(p_3) = 0.9702 \times 0.97 = 0.941094$

• Problem 3 :

<table>
<thead>
<tr>
<th>Day</th>
<th>Sun</th>
<th>Mon</th>
<th>Tue</th>
<th>Wed</th>
<th>Thur</th>
<th>Fri</th>
<th>Sat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daily peak MW</td>
<td>600</td>
<td>1100</td>
<td>1200</td>
<td>1100</td>
<td>1200</td>
<td>1200</td>
<td>800</td>
</tr>
</tbody>
</table>

Using the definition of $LOLP_{\text{week}}$,

$$LOLP_{\text{week}} = \frac{1}{7} \sum_{i=1}^{7} LOLP_i,$$ (8)

So we need to calculate $LOLP(600)$, $LOLP(800)$, $LOLP(1100)$, $LOLP(1200)$. To do this we use information of problem 2 and definition of $LOLP$,

$$LOLP(l) = P\{\sum_i A < l\}$$ (9)

The random variable $\sum_i A$ has in this case the distribution,

$$f(x) = 0.000006\delta(x) + 0.000594\delta(x - 400) + 0.0000294\delta(x - 500) + 0.000194\delta(x - 600)$$
$$+ 0.029106\delta(x - 900) + 0.019206\delta(x - 1000) + 0.009506\delta(x - 1100) + 0.941094\delta(x - 1500)$$ (10)

So using $LOLP(l) = \int_{-\infty}^{l} f(x)dx$ we obtain,

$$LOLP(600) = 0.000006 + 0.000594 + 0.000294 = 0.000894$$
$$LOLP(800) = 0.000894 + 0.000194 = 0.001088$$
$$LOLP(1100) = 0.01088 + 0.029106 + 0.019206 = 0.0494$$
$$LOLP(1200) = 0.0494 + 0.009506 = 0.058906$$ (11)
Finally \( LOLP_{\text{week}} \) is,

\[
LOLP_{\text{week}} = \frac{1}{7} \left\{ 0.000894 + 2 \times 0.0494 + 3 \times 0.058906 + 0.001088 \right\} \frac{\text{days}}{\text{days}} = 0.2775 \frac{\text{days}}{\text{week}}
\] (12)

Weekly \( LOLP \) is expressed in \( \text{days/week} \) that means that the probability of loss of load is 0.2775 days by week or 1 day in 3.6036 weeks.

- **Problem 4**: Without loss of generality we can suppose that the period is 1 day (any other period can be scaled). In this case \( LOLE^d \) will be:

\[
LOLE^d = LOLP(l_{\text{day}}) \frac{\text{days}}{\text{day}}
\] (13)

In the same period of time, the \( LOLE^h \) will be:

\[
LOLE^h = \sum_{i=1}^{24} LOLP(l_i) \frac{\text{hours}}{24\text{hours}}
\] (14)

We can restrict the maximum value of (14) if we use the maximum value of \( LOLP(l_i) \) in the period, that correspond to the peak hourly load:

\[
LOLE^h = \sum_{i=1}^{24} LOLP(l_i) \frac{\text{hours}}{24\text{hours}} \leq \sum_{i=1}^{24} LOLP(l_{\text{peak}}) \frac{\text{hours}}{24\text{hours}} = 24LOLP(l_{\text{peak}}) \frac{\text{hours}}{24\text{hours}}
\] (15)

But \( LOLP(l_{\text{peak}}) \) in this case is equal to \( LOLP(l_{\text{day}}) \) if we use the assumption that the daily load is computed using the peak hourly load and by (13) the value is the same that \( LOLE^d \). Using this fact in (15), we arrive to:

\[
LOLE^h \leq 24LOLE^d
\] (16)

- **Problem 5**:

We use \( T = 1 \) with the different peak-loads, \( l = 600, 800, 1100, 1200 \).

\[
U(600) = (600 - 0)P\{0\} + (600 - 400)P\{400\} + (600 - 500)P\{500\} = 600 \times 0.000006 + 200 \times 0.0000594 + 100 \times 0.000294 = 0.1518 MWh
\] (17)

\[
U(800) = (800 - 0)P\{0\} + (800 - 400)P\{400\} + (800 - 500)P\{500\} = 800 \times 0.000006 + 400 \times 0.0000594 + 300 \times 0.000294 = 0.3306 MWh
\] (18)

\[
U(1100) = (1100 - 0)P\{0\} + (1100 - 400)P\{400\} + (1100 - 500)P\{500\} + (1100 - 600)P\{600\}
\]
\[
\quad \quad \quad + (1100 - 900)P\{900\} + (1100 - 1000)P\{1000\}
\]
\[
= 1100 \times 0.000006 + 700 \times 0.0000594 + 600 \times 0.000294 + 500 \times 0.000194
\]
\[
\quad \quad \quad + 200 \times 0.029106 + 100 \times 0.019206 = 8.4376 MWh
\] (19)

\[
U(1200) = (1200 - 0)P\{0\} + (1200 - 400)P\{400\} + (1200 - 500)P\{500\} + (1200 - 600)P\{600\}
\]
\[
\quad \quad \quad + (1200 - 900)P\{900\} + (1200 - 1000)P\{1000\} + (1200 - 1100)P\{1100\}
\]
\[
= 1200 \times 0.000006 + 800 \times 0.0000594 + 700 \times 0.000294 + 600 \times 0.000194
\]
\[
\quad \quad \quad + 300 \times 0.029106 + 200 \times 0.019206 + 100 \times 0.009506 = 14.3282 MWh
\] (20)