Problem 1

a) We make use of the results in HW 1 to obtain the following $LOLP$ vs. load table

<table>
<thead>
<tr>
<th>load (MW)</th>
<th>$LOLP$</th>
<th>$\ln(LOLP)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>0.000006</td>
<td>-12.024</td>
</tr>
<tr>
<td>500</td>
<td>0.000600</td>
<td>-7.4190</td>
</tr>
<tr>
<td>600</td>
<td>0.000894</td>
<td>-7.0200</td>
</tr>
<tr>
<td>700</td>
<td>0.001088</td>
<td>-6.8230</td>
</tr>
<tr>
<td>1000</td>
<td>0.030194</td>
<td>-3.5000</td>
</tr>
<tr>
<td>1100</td>
<td>0.049400</td>
<td>-3.0080</td>
</tr>
<tr>
<td>1200</td>
<td>0.058906</td>
<td>-2.8320</td>
</tr>
<tr>
<td>1600</td>
<td>1.000000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Below we provide a plot of the $\ln(LOLP)$ vs. load.
b) We know from the lecture notes that

\[ H(x) \triangleq P\{ R < x \} = Ke^{\alpha x} \]

Consider the LOLP value at the load level of 1,200 MW. In order to determine \( \alpha \) we make use of two arbitrary points:

\[ x = 0 : \quad H(0) = Ke^{\alpha 0} = 0.058906 \]
\[ x = -100 : \quad H(-100) = Ke^{\alpha(-100)} = 0.0494 \]

Therefore,

\[ \frac{e^0}{e^{-100\alpha}} = \frac{0.058906}{0.049400} \Rightarrow \alpha = 0.00176 \]

c) After adding the new 300 MW unit, we get a new table of LOLP vs. load values.

<table>
<thead>
<tr>
<th>load (MW)</th>
<th>LOLP</th>
<th>ln(LOLP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>3*10^{-7}</td>
<td>-15.02</td>
</tr>
<tr>
<td>400</td>
<td>6*10^{-6}</td>
<td>-12.02</td>
</tr>
<tr>
<td>500</td>
<td>0.0000357</td>
<td>-10.24</td>
</tr>
<tr>
<td>600</td>
<td>0.0000504</td>
<td>-9.00</td>
</tr>
<tr>
<td>700</td>
<td>0.0000601</td>
<td>-9.72</td>
</tr>
<tr>
<td>800</td>
<td>0.0006244</td>
<td>-7.38</td>
</tr>
<tr>
<td>900</td>
<td>0.0009037</td>
<td>-7.01</td>
</tr>
<tr>
<td>1000</td>
<td>0.0025433</td>
<td>-5.97</td>
</tr>
<tr>
<td>1100</td>
<td>0.0035036</td>
<td>-5.65</td>
</tr>
<tr>
<td>1200</td>
<td>0.0039789</td>
<td>-5.53</td>
</tr>
<tr>
<td>1300</td>
<td>0.0316296</td>
<td>-3.45</td>
</tr>
<tr>
<td>1400</td>
<td>0.0498753</td>
<td>-3.00</td>
</tr>
<tr>
<td>1500</td>
<td>0.0589060</td>
<td>-2.83</td>
</tr>
<tr>
<td>1600</td>
<td>0.1059607</td>
<td>-2.24</td>
</tr>
<tr>
<td>1800</td>
<td>1.0000000</td>
<td>0.00</td>
</tr>
</tbody>
</table>

The modified LOLP vs. load curve is provided below.
From the lecture notes we have proved that the effective load carrying capability (ELCC) of a resource is given by

\[ c_{eff} = \frac{1}{\alpha} \left\{ -\ln\left[ \left(1 - p\right) + pe^{-\alpha c} \right] \right\} \]

At a load level of 1,200 MW we have computed \( \alpha = 0.00176 \). Therefore, for \( c = 300 \text{ MW} \) and \( p = 0.95 \) we obtain

\[ c_{eff} = 280.58 \text{ MW} \]

d) With the application of the DSM program, for load levels greater then 1,100 MW we derive the new LOLP values as follows:
When the load values are below 1100 MW, the \( LOLP \) values remain the same. Since we assume that the DSM program is 100% available, which implies that  \( p_{DSM} = 1.0 \), then the \( ELCC \) of the DSM program is

\[
c_{\text{eff}} = 50 \text{ MW}
\]

Problem 2

The generation and load data for system A are given in lecture 3 (Markov Models for Reliability Evaluation) and 4 (Frequency and Duration Techniques) respectively. The generation data for system B are given in lecture 5 (Reliability of Two-Area Interconnections) and the load data for system B are given below
The availability table for system B is

<table>
<thead>
<tr>
<th>$j$</th>
<th>$\ell_j$</th>
<th>$\alpha_j$</th>
<th>$\lambda_{t,j}$ (1/day)</th>
<th>$\lambda_{l,j}$ (1/day)</th>
<th>$p_j = \alpha_j e (j \neq 0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-</td>
<td>2</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>1</td>
<td>80</td>
<td>0.1</td>
<td>0</td>
<td>2</td>
<td>0.05</td>
</tr>
<tr>
<td>2</td>
<td>96</td>
<td>0.2</td>
<td>0</td>
<td>2</td>
<td>0.10</td>
</tr>
<tr>
<td>3</td>
<td>112</td>
<td>0.5</td>
<td>0</td>
<td>2</td>
<td>0.25</td>
</tr>
<tr>
<td>4</td>
<td>120</td>
<td>0.2</td>
<td>0</td>
<td>2</td>
<td>0.10</td>
</tr>
</tbody>
</table>

We compute the following representative values for the two-area interconnection example given in lecture 5. For system A as an isolated system we obtain

\[
P\{R^4 = -100\} = P\left\{\sum_{i=1}^{4} A_i = 0\right\} P\{L = 100\} + P\left\{\sum_{i=1}^{4} A_i = 50\right\} P\{L = 150\}
\]
\[
= (0.0000026)(0.1*0.5) + (0.0002458)(0.2*0.5)
\]
\[
= 0.000001 + 0.0000246 = 0.0000247
\]

\[
P\{R^4 \leq -100\} = P\{R^4 = -100\} + P\{R^4 = -120\} + P\{R^4 = -140\} + P\{R^4 = -150\}
\]
\[
= 0.0000259
\]

For the frequency computation we define the set

\[
\mathcal{S}(-100) = \{ i : R^4_i \leq -100 \}
\]

then the frequency
For system B as an isolated system we obtain

\[ \mathbb{P}\{\mathcal{S}(\leq -100)\} = \mathbb{P}\{R^B \leq -100\} = \sum_{i \in \mathcal{S}(\leq -100)} p_i \left( \sum_{j \in \mathcal{S}(\leq -100)} \lambda_{ji} \right) = \\
= p^B_{-100} (0.079 + 2) + p^B_{-120} (0.105 + 2) \\
+ p^B_{-140} (0.105 + 2) + p^B_{-150} (2) + p^B_{-100} (0.105 + 2) \\
= 0.0000246 \times 2.079 + 0.0000003 \times 2.105 + \\
+ 0.0000007 \times 2.105 + 0.0000003 \times 2 \\
+ 0.0000001 \times 2.105 = 0.0000541 \\
\]

For system B as an isolated system we obtain

\[ P\{R^B = 120\} = P\left\{ \sum_{i=1}^{3} A^B_i = 120 \right\} P\{L = 0\} = 0.0737280 \times 0.5 = 0.036864 \\
\]

\[ P\{R^B \leq 120\} = \sum_{i^B \leq 120} P\{R^B \leq r^B_i\} = 0.557632 \\
\]

\[ \mathbb{P}\{R^B \leq 120\} = p^B_{120} (0.026301) + p^B_{80} (2) + p^B_{64} (2) + p^B_{48} (2) + \\
+ p^B_{40} (2) = 0.8857 \\
\]

For the interconnected systems A and B with unconstrained tie-line capacity we can derive the probabilities of any of the failure states as follows

\[ P\{R^A = r^A_i \text{ and } R^B = r^B_i\} = P\{R^A = r^A_i\} P\{R^B = r^B_i\} \text{ due to statistical independence of the two areas. Hence,} \\
\]

\[ P\{R^A = -20 \text{ and } R^B = 160\} = P\{R^A = -20\} P\{R^B = 160\} \\
= 0.0008847 \times 0.442368 = 0.000391 \]
Problem 3

i)

We know that

\[
P \{ R^v < 0 \} = P \{ R^v + A^{AB} < 0 \} = P \{ R^v < 0 \} + P \{ A^v < 0 \} \]

and,

\[
P \{ R^v + A^{AB} < 0 \} = \sum_{i,j} p_{ij}
\]

with states \( i, j \in \{ R^v + 50 < 0 | R^v \geq 50 \} \cup \{ R^v + 20 < 0 | 0 < R^v < 50 \} \cup \{ R^v < 0 | R^v \leq 0 \}
\]

From the table in slide 29, lecture 5 we obtain by adding all the values below the red staircase line plus the values above the staircase for \( r_i^v \leq -70 \) (that give negative reserves when we add the 50 MW assistance from system B). Hence,

\[
P \{ R^v + A^{AB} < 0 \} = 0.6031 \times 10^{-3}
\]

Consequently,

\[
P \{ R^v < 0 \} = 0.999 \left( 0.6031 \times 10^{-3} \right) + 0.001 \left( 0.0041052 \right) = 0.000607
\]

For \( c_i = 100 \text{ MW} \) we can derive in a similar way

\[
P \{ R^v + A^{AB} < 0 \} = \sum_{i,j} p_{ij} = 0.000548
\]

with states \( i, j \in \{ R^v + 100 < 0 | R^v \geq 100 \} \cup \{ R^v + 20 < 0 | 0 < R^v < 100 \} \cup \{ R^v < 0 | R^v \leq 0 \}
\]

Therefore,

\[
P \{ R^v < 0 \} = 0.999 \left( 0.000548 \right) + 0.001 \left( 0.0041502 \right) = 0.000552
\]

ii) For the frequency evaluation we proceed as follows
The total frequency contribution of this case is given by

\[
\sum_j p_{jA} \mathbb{F}\{ R^A < -A_j \} = 0.001241
\]

The total contribution of the second case is
Hence,

\[
\sum_i p_i^A \mathbb{F} \{ A_{AB}^A < - r_i^A \} = 0.001094
\]

\[
\mathbb{F} \{ R^A < 0 \mid A_j = 50 \} = 0.001241 + 0.001094 = 0.002335
\]

Therefore the total contribution is computed by the summation

\[
\sum_i p_i^A \mathbb{F} \{ A_{AB}^A < - r_i^A \} = 0.001127
\]
\[
\sum_i p_i^A \mathbb{P}\{ A_{AB}^i < -r_{i}^A \} = 0.001088
\]
\[
\mathbb{P}\{ R^A < 0 \mid A_i = 100 \} = 0.001127 + 0.001088 = 0.00215
\]
Therefore,
\[
\mathbb{P}\{ R^A < 0 \} = 0.00215(0.999) + 0.0084239(0.001) + (0.00274)(0.999)0.0035573 = 0.002166
\]

**Problem 4**

For the evaluation of the probability and frequency that system B fails, we follow exactly the same procedure as with system A. Therefore,

\[
P \{ R^B < 0 \} = P \{ R^B + A^B < 0 \mid A_i = c_i \} p_i + P \{ R^B < 0 \mid A_i = 0 \} (1 - p_i)
\]

We can evaluate each term in \( P \{ R^B < 0 \} \) as follows

\[
P \{ R^B < 0 \} = \sum_{i,j} p_{ij} \quad \forall i, j \in \{ r_j^B < 0 \}
\]
\[
P \{ R^B + A^B < 0 \mid A_i = c_i \} = \sum_{i,j} p_{ij} \quad \forall i, j \in \{ r_i^B \geq c_i \} \cap \{ r_i^A + r_j^B \geq 0 \}
\]

For the frequency evaluation, we know that system B fails

- if the tie line is operating, system B fails due to the fact that system B transitions into a state with \( R^B < 0 \) as a result of unit failures or a load increase in system B and system A provides insufficient assistance to restore area B into a positive margin state. For state \( j \) of system B, we can express the total frequency contribution of this situation as

\[
p_i \sum_i p_i^A \mathbb{P}\{ R^B + r_i^A < 0 \} \quad \forall i \in \{ 0 \leq r_i^A = R^A \leq c_i \}
\]

- If system B receives assistance from system A, but system A transitions to a state of lower reserves (outages of its units or increase in its load) corresponding to a state with \( R^B < 0 \), we express the frequency contribution of this case as
If system B receives assistance from system A and the tie line fails, we can write the frequency contribution of this situation as

\[ p_i \sum_j p_j^n \mathcal{F}\{R^A + r_j^B < 0\} \quad \forall j \in \{r_j^B < 0\} \]

- If system B receives assistance from system A and the tie line fails, we can write the frequency contribution of this situation as

\[ \lambda_i p_i \left( \sum_{i,j} p_{ij} \right) \quad \forall i, j \in \{R^B < 0\} \cap \{R^B + A^A \geq 0\} \text{ with } 0 \leq A^A \leq c_i \]

1. If the tie line is not operating, system B fails if it transitions to a state with \( R^B < 0 \). We can write its frequency contribution as

\[ (1 - p_i) \mathcal{F}\{R^B < 0\} \]

The total failure frequency is expressed as the sum of the frequencies of all the previous cases. Therefore,

\[ \mathcal{F}\{R^B < 0\} = p_i \left( \sum_i p_i \mathcal{F}\{R^A + r_i^B < 0\} + \sum_j p_j^n \mathcal{F}\{R^A + r_j^B < 0\} + \lambda_i \sum_{i,j} p_{ij} \right) + (1 - p_i) \mathcal{F}\{R^B < 0\} \]