**University of Illinois** Fall 2018

# ECE 586GT: Exam II

Monday, December 10, 2018
7:00 p.m. — 8:30 p.m.
2013 Electrical Engineering Building

**Name:** ________________________________

Instructions

This exam is closed book and closed notes except that three 8.5" ×11" sheets of notes are permitted; both sides may be used. Calculators, laptop computers, PDAs, iPods, cellphones, e-mail pagers, headphones, etc. are not allowed.

Grading

1. 40 points ____________
2. 30 points ____________
3. 30 points ____________

Total (100 points) ____________

1. **[40 points]** Recall the standard model for infinite repeated play of the prisoners' dilemma game:

| | C | D |
|---|---|---|
| C | 1,1 | -1,2 |
| D | 2,-1 | 0,0 |

with payoff of player $i$ given by $J_i = (1-\delta)\sum_{t=1}^{\infty} g_i(s_t)\delta^{t-1}$ for some $\delta \in (0,1)$. Let $s^T = (s_1^T, s_2^T)$ denote the profile of trigger strategies, such that $s_i^T(h_t) = C$ if $h_t = ((C,C),\ldots,(C,C))$ and $s_i^T(h_t) = D$ otherwise. Consider the following variation, for some $\epsilon \in (0,1)$. If a player attempts to play action $C$ in a stage $t$, due to a communication error, the player's publicly announced action is $D$ with probability $\epsilon$. If a player attempts to play action $D$ no error is possible, so the publicly announced action for the player is also $D$. If both players play $C$ the errors happen independently for the two players. The payoffs of the players are based on the publicly announced actions. When $s^T$ is used, it is applied to the history of publicly announced actions (not on the actions attempted by the players). The goal of this problem is to find conditions on $\epsilon$ and $\delta$ such that $s^T$ is subgame perfect.

(a) Suppose both players follow $s^T$ with one exception: namely, player 1 deviates in the first stage. What is the expected payoff of player 1? (Your answer should depend on $\epsilon$ and $\delta$.)

(b) Suppose both players follow $s^T$. Let $X$ denote the first stage such that at least one player is publicly announced to have played $D$. What are $P\{X > t\}$, $P\{X = t\}$, and $\alpha \triangleq E[g_i(s_t^T)|X = t]$ for $t \geq 1$?

(c) For what values of $\delta$ and $\epsilon$ is $s^T$ a subgame perfect equilibrium? (Due to time restrictions, you can leave your answer in terms of infinite sums.)

(d) What is $\lim_{\delta \to 1} J_i(s^T)$ for either player $i$ and a fixed $\epsilon \in (0, 1)$? Explain. (Hint: This can be answered independently of parts (a) - (c).)

2. **[30 points]** Consider the sale of $k$ identical objects to $n$ bidders such that $1 \leq k \leq n-1$. Suppose each bidder can be allocated at most one object, and bidder $i$ has value $v_i$ for any one of the objects.

   (a) Describe the VCG mechanism for selling the objects. In other words, describe the VCG allocation rule and associated payment rule. Explain how your solution follows from the general form of VCG mechanisms, and simplify it as much as possible.

   (b) Describe the revenue optimal, incentive compatible, individually rational selling mechanism for selling the objects, under the assumption that the valuations are independent and uniformly distributed over the interval $[0, 1]$. To be definite, suppose that the seller must sell all $k$ objects, or equivalently, the value to seller of any object is $-\infty$. Explain how your solution follows from the general form of revenue optimal mechanisms, and simplify it as much as possible.

3. **[30 points]** The cooperative games below are for a set of four players, $I = \{1, 2, 3, 4\}$. As usual, assume transferrable utilities. Simplify your answers as much as possible.

(a) (10 points) Suppose $v_1 = (v_1(S))_{S \subset I}$ is given by

$$v_1(S) = \begin{cases} 1 & \text{if } \{1,2\} \subset S \text{ or } \{3,4\} \subset S \\ 0 & \text{else} \end{cases} .$$

Is $(I, v_1)$ cohesive? Find the core of $(I, v_1)$.

(b) (20 points) Suppose $v = v_1 + v_2$, where $v_1$ is defined in (a), and

$$v_2(S) = \begin{cases} 1 & \text{if } \{1,3\} \subset S \text{ or } \{2,4\} \subset S \\ 0 & \text{else} \end{cases} .$$

For example, the players could be working for two days, with values $v_1$ the first day and $v_2$ the second day. Is $(I, v)$ cohesive? Find the core of $(I, v)$.