University of Illinois Fall 2018

ECE 586GT: Exam I

Thursday, October 18, 2018
7:00 p.m. — 8:30 p.m.
2013 Electrical Engineering Building

1. **[42 points]** Consider the two-player zero sum game in which player one, the row selector, seeks to minimize ℓ and player 2, the column selector, seeks to maximize ℓ, where ℓ is given by the following table:

		Player 2		
		1	2	3
Player 1	1	0	1	2
	2	1	0	1
	3	2	1	0

As usual, pure strategies are special cases of mixed strategies.

   (a) (6 points) Identify all minmax optimal pure strategies for player 1.
   (b) (6 points) Identify all maxmin optimal pure strategies for player 2.
   (c) (12 points) Identify all maxmin optimal mixed strategies for player 2.
   (d) (12 points) Identify all minmax optimal mixed strategies for player 1.
   (e) (6 points) Identify the maximum expected payoff to player 2 over all correlated equilibria (Hint: Can be done with no calculation beyond what you did for parts (c) or (d).)

2. **[30 points]** Consensus games (Coles and Olives, 1980). Consider a normal form game with finite set of players I and action sets $S_i = \{0, 1\}$ for all $i \in I$. For each player i let A_i, with $A_i \subset I \backslash \{i\}$. For a strategy profile $s = (s_i)_{i \in I}$, let $u_i(s) = \sum_{j \in A_i} \mathbf{1}_{\{s_i = s_j\}}$. In other words, the payoff of player i is the number of players in A_i with which the player agrees.

   (a) (15 points) Suppose the sets $(A_i)_{i \in I}$ are neighborhood sets for an undirected graph. In other words, suppose for each $i, j \in I$, $i \in A_j$ if and only if $j \in A_i$. Is the game necessarily a potential game? If so, identify the potential function. If not, give an example and argue why a potential function does not exist for it.
   (b) (15 points) Repeat part (a), but without the assumption $i \in A_j$ if and only if $j \in A_i$.

3. **[28 points]** Determine whether each statement is TRUE or FALSE, *and give a justification for your answer* for more than half credit.

   (a) $x = 0.5$ is a stable equilibrium for the differential equation $\dot{x}_t = x_t(1 - x_t)(x_t - 0.5)^2$.
   (b) Suppose the sequence of strategy profiles produced by iterated best response for some finite normal form game is periodic with period 2, alternating between strategy profile vectors $s^{(0)}$ and $s^{(1)}$ in $S = S_1 \times \cdots \times S_n$. Let s^* be the mixed strategy profile such that, $s_i^* = s_i^{(0)}$ with probability one half, and $s_i^* = s_i^{(1)}$ with probability one half, with the choices being made independently for different players. Then s^* must be a Nash equilibrium vector.
   (c) Consider a two player game with vector valued payoffs and suppose sets S_1 and S_2 are both approachable by player 1. Then $S_1 \cup S_2$ must be approachable by player 1.
   (d) Consider a two player game with vector valued payoffs and suppose sets S_1 and S_2 are both approachable by player 1. Then $S_1 \cap S_2$ must be approachable by player 1.