Problem 1

(10 pts.) Show that $SPACE(n) \neq NP$.

Problem 2

Recall that $E = DTIME(2^{O(n)})$ is the class of problems solvable by deterministic turing machine in time $2^{O(n)}$, where $n$ is the length of the input. We say that a language $A$ has a many-to-one polynomial time reduction to a language $B$, written $A \leq_{m^p} B$ if there is a polynomial time computable function $f(\cdot)$ such that for every instance $x$ we have $x \in A \Leftrightarrow f(x) \in B$.

- (10 pts.) Show that $NP$ is closed under polynomial many-to-one reductions, that is $A \leq_{m^p} B$ and $B \in NP$ implies $A \in NP$.
- (10 pts.) Show that if $E$ were closed under many-to-one reductions, we would have a contradiction to the time hierarchy theorem. Conclude that $NP \neq E$.

Problem 3

(20 pts.) Define a language $L$ which belongs to $SIZE(O(1))$ and is undecidable.

Problem 4

Recall that $EXP = DTIME(2^{n^{O(1)})}$.

- (15 pts.) Prove that if $P = NP$ then $\Sigma_k = P$ for all $k$.
- (15 pts.) Prove that if $P = NP$ then there is a problem in $EXP$ that requires circuits of size $2^{\Omega(n)}$.

Problem 5

(20 pts.) Prove that if $NP \subseteq BPP$ then $NP = RP$. 