ECE 573 – Power System Operations and Control

5. Review of Economic Dispatch

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ECONOMIC DISPATCH \((ED)\)

- The economic dispatch \((ED)\) function serves to allocate the total generation among the committed units so as to minimize the costs to serve the system load and losses, with the scheduled interchanges and specified operational and equipment constraints fully taken into consideration.
ECONOMIC DISPATCH (ED)

- The ED function came into use in the context of the vertically integrated utility structure and involves the solution of a least-cost problem from the system operator's point of view.

- The aim of this brief review is to provide insights into the ED decision and examine the solution schemes of ED problem formulations with varying level of detail.
SALIENT CHARACTERISTICS OF $ED$

- $ED$ is solved for a **snapshot in time** for a given system state: typically, this state is assumed to hold constant over a specified period.

- $ED$ objective: determination of least production costs for system operations.

- $ED$ problem formulation may include:
  - transmission system losses
  - each generating unit’s economics
  - scheduled interchanges
ECONOMIC DISPATCH: KEY CHARACTERISTICS

- total system load
- equipment limitations
- network effects

The ED problem typically neglects

- reactive flow/voltage aspect representation
- line flow constraints
- local area generation constraints
- security considerations
THERMAL UNIT INPUT – OUTPUT CURVE

**input**

MMBtu/h

**output**

MW

**minimum capacity**

**maximum capacity**
THERMAL UNIT HEAT RATE

\[ \text{heat rate} = \frac{\text{input}}{\text{output}} \]

\[ \text{incremental heat rate} = \frac{\text{incremental input}}{\text{incremental output}} \]

Minimum capacity

Maximum capacity

A possible operating point
INCREMENTAL INPUT–OUTPUT CURVE

**INCREMENTAL HEAT RATE**

\[
\Delta h = \Delta p
\]

**HEAT RATE**

\[
\frac{\Delta}{\Delta p} \frac{h}{p}
\]

**INCREMENTAL HEAT RATE**

\[
\frac{\Delta h}{\Delta p} = \text{incremental input output}
\]

**INPUT**

MMBtu/h

**OUTPUT**

MW
INPUT – OUTPUT MEASUREMENTS

heat input
\((\text{MMBtu/h})\)

output
\((\text{MWh/h})\)

measurement
heat content & flow - rate of fuel

measurement
energy output

set control valve points
EXAMPLE:
CWLP DALLMAN UNITS 1 and 2

heat input (MMBtu/h)

| 972 | 901 | 835 | 773 | 715 | 659 | 605 | 552 | 499 | 446 | 392 | 336 |

output (MWh/h)

| 80  | 75  | 70  | 65  | 60  | 55  | 50  | 45  | 40  | 35  | 30  | 25  |
CWLP DALLMAN UNITS 1 and 2
INPUT – OUTPUT CURVE FITTING

$\text{MMBtu/h}$

$\text{MWh/h}$

- $200$
- $400$
- $600$
- $800$
- $1,000$

- $25$
- $30$
- $35$
- $40$
- $45$
- $50$
- $55$
- $60$
- $65$
- $70$
- $75$
- $80$
CWLP DALLMAN UNITS 1 and 2
HEAT RATE AND IHR CURVES

**MMBtu/MWh**

- **Heat rate**
- **Incremental heat rate**

**MWh/h**

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The input-output curve is used to determine the $/h$ generator costs of production. We use the notation of $C_i(P_i)$ to designate the cost function at generation level $P_i$ for generator $i$; this information is typically available directly from the input-output plot or simple scaling of the input-output curve.
GENERATING UNIT ECONOMICS

- $C_i(P_i)$ is assumed to be a monotonically non-decreasing function over the interval

\[ P_i^{\text{min}} \leq P_i \leq P_i^{\text{max}} \]

- The incremental input-output curve is used to determine the \textit{marginal costs} $\frac{dC_i(\cdot)}{dP_i}$, typically called the \textit{incremental costs} of generator $i$
Marginal costs are defined as the rate of change of the costs with respect to a change in the load: in effect, we think of marginal costs as the costs to generate the last $MWh$ at a specified loading or demand level.
We often think of the marginal costs as the costs to generate one additional $MWh$

In this context, incremental costs and marginal costs are identical quantities

We can define decremental costs in a similar manner
A simplified model is used and the following assumptions are imposed:

- the system consists of $n$ generating units at a single node with a point mass load
- all network effects are ignored
- no losses are represented
The problem formulation is simply

\[
\min f(x) = \sum_{i=1}^{n} C_i(P_i)
\]

\[
\text{s.t.} \quad \sum_{i=1}^{n} P_i = P_D
\]

with

\[
x \triangleq \left[ P_1, P_2, \ldots, P_n \right]^T
\]

and \(P_D\) represents the total system demand
This equality constrained minimization problem has a single constraint and so $\lambda$ is simply a scalar variable in the Lagrangian function.

We write the Lagrangian for \(CED\)

$$\mathcal{L}(x, \lambda) = \sum_{i=1}^{n} C_i(P_i) + \lambda \left( P_D - \sum_{i=1}^{n} P_i \right)$$
The necessary conditions for optimality require

\[ \nabla_x \mathcal{L} = \begin{bmatrix} \frac{\partial C_1}{\partial P_1} - \lambda, \frac{\partial C_2}{\partial P_2} - \lambda, \ldots, \frac{\partial C_n}{\partial P_n} - \lambda \end{bmatrix} = 0^T \]

\[ \nabla_\lambda \mathcal{L} = P_D - \sum_{i=1}^{n} P_i = 0 \]

Therefore at the optimum

\[ \frac{\partial C_i}{\partial P_i} = \lambda \quad i = 1, 2, \ldots, n \]

\[ \sum_{i=1}^{n} P_i = P_D \]

coordination equations
ELECTRICITY MARKET \( \lambda \) DISPATCH

\[
\frac{\text$/MWh}}{\text{system } \lambda}} \quad \text{MW}
\]

\[P_1 \quad P_2 \ldots \quad P_n\]
Step 0: pick a value $\lambda^{(0)}$ for $\lambda$ and set $v = 0$; select a value for $\alpha$, the acceleration factor.

Step 1: for $i = 1, 2, ..., n$, determine $P_i^{(v)}$ such that

$$
\frac{dC_i \left( P_i^{(v)} \right)}{dP_i} = \lambda^{(v)}
$$
\[ \lambda - \text{ITERATION SCHEME FOR CED} \]

Step 2: if

\[ \sum_{i=1}^{n} P_i^{(v)} - P_D = 0 \]

stop; else set

\[ \lambda^{(v+1)} = \lambda^{(v)} + \alpha \left( P_D - \sum_{i=1}^{n} P_i^{(v)} \right) \]

and return to Step 1
The simplified model is modified to include the generation unit limits:

\[ P_{i_{\text{min}}} \leq P_i \leq P_{i_{\text{max}}} \quad i = 1, 2, \ldots n \]

The inequality constrained classical economic dispatch problem is formulated as the equality and inequality constrained program.
We can modify the $\lambda$ – iteration scheme to take into account the generation unit limit: essentially,

\[
\begin{align*}
\min \quad & f(x) = \sum_{i=1}^{n} C_i(P_i) \\
\text{s.t.} \quad & \sum_{i=1}^{n} P_i - P_D = 0 \\
& P_{i\min} \leq P_i \leq P_{i\max} \quad i = 1, 2, \ldots, n
\end{align*}
\]
INEQUALITY – CONSTRAINED CED

we fix the generator output at the violated limit and remove that generator from all the subsequent \( \lambda \) – iterations; in this modified approach, the equal \( \lambda \) criterion applies only to the generators that are within their limits and not at either limit.

- Alternatively, we may introduce penalty functions to handle the generation limits and append them to the objective function.
We define the penalty functions $\kappa_i^{\text{max}} p_i^{\text{max}}(x)$ and $\kappa_i^{\text{min}} p_i^{\text{min}}(x)$ for each generator $i$ limit constraint

$$p_i^{\text{max}}(x) \triangleq \begin{cases} 0 & \text{if } P_i \leq P_i^{\text{max}} \\ (P_i - P_i^{\text{max}})^2 & \text{if } P_i > P_i^{\text{max}} \end{cases}$$

$$p_i^{\text{min}}(x) \triangleq \begin{cases} 0 & \text{if } P_i \geq P_i^{\text{min}} \\ (P_i^{\text{min}} - P_i)^2 & \text{if } P_i < P_i^{\text{min}} \end{cases}$$
The equality constraint

\[ \sum_{i=1}^{n} P_i = P_D \]

may be replaced by the two inequalities

\[ \sum_{i=1}^{n} P_i \leq P_D \quad \text{and} \quad \sum_{i=1}^{n} P_i \geq P_D \]

We introduce into the objective function the corresponding penalty terms
INEQUALITY – CONSTRAINED CLASSICAL \textit{ED}

\[ \kappa_i^+ \left( \sum_{i=1}^{n} P_i - P_D \right)^2 \quad \text{and} \quad \kappa_i^- \left( P_D - \sum_{i=1}^{n} P_i \right)^2 \]

penalty coefficients

or simply \( \kappa \left( \sum_{i=1}^{n} P_i - P_D \right)^2 \)

\[ \square \quad \text{We thus transform the (ICCED) into the} \]
unconstrained minimization problem

\[
\min_{\mathbf{x}} \hat{f}(\mathbf{x}) = \sum_{i=1}^{n} C_i (P_i) + \kappa \left( \sum_{i=1}^{n} P_i - P_D \right)^2 + \\
+ \sum_{i=1}^{n} \left[ \kappa_i^{\text{max}} p_i^{\text{max}}(\mathbf{x}) + \kappa_i^{\text{min}} p_i^{\text{min}}(\mathbf{x}) \right]
\]

We then solve the UMP; typically, heuristics help in setting the values of the $\kappa$ coefficients
NEWTON SOLUTION SCHEME

- We evaluate the gradient and Hessian of $\hat{f}(x)$

- The $i^{th}$ component of $\nabla \hat{f}$ is given by

$$\frac{\partial \hat{f}}{\partial P_i} = \frac{\partial C_i}{\partial P_i} + 2\kappa \left( \sum_{i=1}^{n} P_i - P_d \right) + \kappa_{i}^{\text{max}} \frac{\partial p_{i}^{\text{max}}}{\partial P_i} + \kappa_{i}^{\text{min}} \frac{\partial p_{i}^{\text{min}}}{\partial P_i}$$

with

$$\frac{\partial p_{i}^{\text{max}}}{\partial P_i} = \begin{cases} 0 & \text{if } P_i \leq P_{i}^{\text{max}} \\ 2(P_i - P_{i}^{\text{max}}) & \text{if } P_i > P_{i}^{\text{max}} \end{cases}$$

$$\frac{\partial p_{i}^{\text{min}}}{\partial P_i} = \begin{cases} 0 & \text{if } P_i \geq P_{i}^{\text{min}} \\ -2(P_{i}^{\text{min}} - P_i) & \text{if } P_i < P_{i}^{\text{min}} \end{cases}$$
NEWTON SOLUTION SCHEME

The element $i, j$ of the Hessian $H(x)$ is given by

$$H_{i,j} = \begin{cases} 
2\kappa & i \neq j \\
\frac{\partial^2 C_i}{\partial P_i \partial P_i} + 2\left[\kappa + \kappa_i^{\text{max}} + \kappa_i^{\text{min}}\right] & i = j
\end{cases}$$

Whenever $P_i^{\text{min}} \leq P_i \leq P_i^{\text{max}}$ this term is 0

Recall that

$$H_{i,j} = H_{j,i} , i \neq j$$
NEWTON ALGORITHM

Step 0: initialize \( \bar{x}^{(0)} = \left[ P_1^{(0)}, P_2^{(0)}, \ldots, P_n^{(0)} \right]^T, \kappa^{(0)}, \kappa_{i}^{\max(0)} \) and \( \kappa_{i}^{\min(0)}, i = 1, 2, \ldots, n \); set \( \nu = 0 \) and specify the values of the tolerance and the penalty multiplier \( d \).

Step 1: compute \( \nabla \hat{f} \left( x^{(\nu)} \right) \) and \( H \left( x^{(\nu)} \right) \).

Step 2: if \( \left\| \nabla \hat{f} \left( x^{(\nu)} \right) \right\| < \varepsilon \), stop; else, solve for \( \Delta x \).
NEWTON ALGORITHM

\[ \mathbf{H}(\mathbf{x}^{(v)}) \Delta \mathbf{x} = -\left[ \nabla \hat{f}(\mathbf{x}^{(v)}) \right]^T \]

Step 3: evaluate stepsize \( \alpha^{(v)} \) and compute

\[ \mathbf{x}^{(v+1)} = \mathbf{x}^{(v)} + \alpha^{(v)} \Delta \mathbf{x} \]

Step 4: check the constraint violations and compute the new values for the coefficients whenever needed:
NEWTON ALGORITHM

\textit{a}) for each inequality constraint with

\[ \left| \text{violation}^{(v)} \right| > \left| \text{violation}^{(v-1)} \right| \]

set the corresponding

\[ \kappa^{\text{min}} = \kappa^{\text{min}} \cdot d \quad \text{or} \quad \kappa^{\text{max}} = \kappa^{\text{max}} \cdot d \]

\textit{b}) for the equality constraint with

\[ \left| \sum_{i=1}^{n} P_i^{(v)} - P_D \right| > \left| \sum_{i=1}^{n} P_i^{(v-1)} - P_D \right| \]

set \( \kappa = \kappa \cdot d \) and return to Step 1
The simplest representation of network effects is the representation of the losses $P_{loss}$ in terms of the decision variable $x$.

The inclusion of losses modifies the equality constraint

$$\sum_{i=1}^{n} P_i - P_D - P_{loss}(x) = 0$$
The coordination equations become

\[
\frac{\partial C}{\partial P_i} \left( 1 - \frac{\partial P_{\text{loss}}}{\partial P_i} \right)^{-1} = \lambda
\]

\[
\sum_{i=1}^{n} P_i - P_{\text{loss}}(x) = P_D
\]

The term \( \left( 1 - \frac{\partial P_{\text{loss}}}{\partial P_i} \right)^{-1} \) is called the *penalty factor* of generator \( i \).

There are various ways of representing \( P_{\text{loss}}(x) \) and of evaluating the penalty factors.
A common representation of system losses is by a quadratic function expressed in terms of the so-called $B$ coefficients

$$P_{loss}(x) = B_0 + \sum_{i=1}^{n} B_i P_i + \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij} P_i P_j, \quad B_{ij} = B_{ji} \text{ for } i \neq j$$

The $B_{ij}$ coefficients are determined from multiple power flows; typically, there are seasonal $B_{ij}$ coefficients to distinguish the various regimes under which the system may operate.