17. Optimal Offer Strategies in Competitive Electricity Markets
OUTLINE

- The competitive electricity market \((CEM)\)
- Construction of the \(CEM\) framework
- Key features of the framework
- Formulation of optimal offer strategies
- Numerical results
A key aspect of the electricity restructuring in England and Wales was the establishment of the England and Wales Power Pool (EWPP) which played a very important role in enabling the development of competition in England and Wales. The EWPP became the basis for the pool paradigm.
BACKGROUND

- The *EWPP* operator runs the *spot electricity markets* and provides the centralized scheduling and dispatch of generation to meet the load around the clock.

- Virtually all power was transacted through the *EWPP*: the Pool operator buys all the suppliers’ sales and sells all the buyers’ purchases.
THE PROBLEM

- We consider the \textit{CEM} with many buyers and many sellers.
- The \textit{commitment and dispatch} of units are based on the outcomes of a competitive auction procedure.
- Each market player submits a sealed offer/bid stating the price and quantity at which he is willing to sell/buy power.
- \textbf{Buyers} may be large customers, brokers, distribution entities or perhaps generators who...
THE PROBLEM

choose to buy in the *CEM* rather than generate their own electricity or to purchase under longer-term contracts

- **Sellers** are generators and brokers/marketers
- Generators are entities that own resources and/or can sell energy/capacity to the *CEM*
- The *CEM* dispatcher determines the *least-cost* commitment and dispatch to meet the demand
THE CEM STRUCTURE

seller 1

seller i

seller M

MWh

seller

buyer 1

buyer j

buyer N

$MWh

$MWh

$MWh

CEM operator

$MWh

$MWh

$MWh

$
THE EWPP SEQUENCE OF EVENTS

- **load forecast**
- **availability declaration period (ADP)**
- **schedule day (SD)**

**Day 0**
- 10:00 a.m.
- 9:00 p.m.

**Day 1**
- 5:00 a.m.

**Day 2**
- 5:00 a.m.
- 12:00 noon

**Sellers submit offer files**
THE OFFER FILE

- Each *genset* – a unit or group of units, which are considered together, for dispatch purposes – submits an *offer file*

- The offer file contains information on:
  - *availability* of the genset for the *ADP* (maximum power offered by the seller for each subperiod)
  - *offer price* of the *genset* (need not be related to the costs)
  - *start-up price* of the *genset* and the seller must also provide unit operational information for the use of the *CEM* operator
EXAMPLE OF A WILLANS LINE

offer price to the CEM in $/h

$ b^0$

$ p^\text{min}$

$ p^\text{max}$

$ \eta^1$

$ \eta^2$

$ \eta^3$

$ \varepsilon^1$

$ \varepsilon^2$

MW
Forecasts demand

Collects the offers/bids from sellers/buyers

Must be financially independent of the sellers and the buyers

Ensures secure and efficient system operations

Determines the least-price commitment and dispatch by solving

- a problem akin to $UC$

- using as costs the offer/bid prices
OFFER DEFINITION

- **Offer variable price**: price below which the seller is unwilling to sell
- **Offer start-up price**: price charged on each occasion the seller’s unit is started up
- **Offer capacity**: the capacity offered for dispatch for each period of the *schedule horizon*
THE AUCTION MECHANISM

- Each offer/bid is submitted in sealed-bid form; the CEM operator determines the accepted offers/bids.
- Competition among the sellers is for the right to serve load; each accepted offer is paid a price that is at least as high as its offer price.
- All sellers are paid a uniform price based, typically, on the price of the highest accepted offer/lowest accepted bid to clear the market.
DEFINITIONS

- The period $t$ system marginal price ($SMP$) is the rate of change of the total costs incurred by the $CEM$ with respect to a change in demand in period $t$, with demand in each other time period remaining unchanged.
DEFINITIONS

- The period $t$ system reserves price ($SRP$) is the rate of change of the total costs incurred by the $CEM$ with respect to a change in the reserves requirements for period $t$, with reserves requirements remaining unchanged in all other time periods.
THE SYSTEM MARGINAL PRICE

system marginal price

$/MWh

MW

genset incremental prices

price of the most expensive genset dispatched

forecasted price – insensitive demand

ECE 573 © 2001 - 2015 George Gross, University of Illinois at Urbana-Champaign; All Rights Reserved
The seller must determine the values of the three offer variables:

- availability (maximum capacity);
- variable price; and,
- start-up price

for each period so as to maximize its profits over the scheduling horizon.

The prices offered need have no relationship to the actual costs of the unit operations.
GENERATOR UNIT COSTS

- **Variable costs**: fuel and variable O&M costs for unit operations – dispatch costs

- **Start-up costs**: costs incurred for the unit start-up – commitment costs

- The **fixed investment costs are completely ignored** in the analysis since they must be paid independent of the operating policy and the incurred variable costs; these are, in effect, *sunk costs* and are not under the control of the decision maker in the offer formulation problem.
OFFER FORMULATION PROBLEM

Specify the offer to maximize profits given
- the generating resource capability
- the variable costs
- the unit physical constraints on:
  - the minimum up and down times
  - the min/max generation levels
  - the reserves contribution limits

The optimization is undertaken under *imperfect information* on the
- system load
- the actions of the competing sellers
THE OFFER SPECIFICATION

- The components of the seller $i$ offer are
  - $b_i^F(\cdot)$: the offer variable price function;
    
    $b_i^F(p_i)$ is the price in $$/h$ at which seller $i$ is willing to sell $p_i$ MW in the CEM.
  
  - $b_i^S(\cdot)$: the offer start-up price function and is allowed to depend purely on the downtime.
THE OFFER SPECIFICATION

\( \vdash a_i : \) the \( T \)-dimensional offer capacity

vector \[ \begin{bmatrix} a_{i1}, a_{i2}, \ldots, a_{it}, \ldots, a_{iT} \end{bmatrix}^T \]

with the \( t^{th} \) component \( a_{it} \geq 0 \) representing the

maximum offered capacity to the CEM

for the period \( t, 1 \leq t \leq T \)

\[ \square \] The offer of seller \( i \) is defined by the triplet

\[ \beta_i \triangleq \{ b_i^F(\cdot), b_i^S(\cdot), a_i \} \]
NOTATION

\( t \) = time period index with \( t = 1, 2, \ldots, T \)

\( \Delta \) = duration of each time period

\( D_t \) = system demand in period \( t \)

\( R_t \) = system reserves requirements in period \( t \)

\( u_{i,t} \) = status of unit \( i \) in period \( t \)

\[ u_{i,t} = \begin{cases} 1 & \text{unit on} \\ 0 & \text{unit off} \end{cases} \]

\( p_{i,t} \) = real power output of unit \( i \) in period \( t \)
NOTATION

\[ r_{i,t} = \text{reserves contribution of unit } i \text{ in period } t \]

\[ \tau_{i,t} = \text{down time of unit } i \text{ at the end of period } t \]

\[ D = \begin{bmatrix} D_1, D_2, \ldots, D_T \end{bmatrix}^T \]

\[ R = \begin{bmatrix} R_1, R_2, \ldots, R_T \end{bmatrix}^T \]

\[ u_i = \begin{bmatrix} u_{i1}, u_{i2}, \ldots, u_{iT} \end{bmatrix}^T \]

\[ p_i = \begin{bmatrix} p_{i1}, p_{i2}, \ldots, p_{iT} \end{bmatrix}^T \]

\[ r_i = \begin{bmatrix} r_{i1}, r_{i2}, \ldots, r_{iT} \end{bmatrix}^T \]
\[ u_i = \left[ u_{1i}, u_{2i}, \ldots, u_{Mi} \right]^T \]

\[ p_i = \left[ p_{1i}, p_{2i}, \ldots, p_{Mi} \right]^T \]

\[ r_i = \left[ r_{1i}, r_{2i}, \ldots, r_{Mi} \right]^T \]

\[ \Sigma_i = \left\{ u_i, p_i, r_i \right\} = \text{unit } i \text{ operating schedule} \]

\[ \Sigma = \left\{ u, p, r \right\} = \text{the system operating schedule} \]
THE CEM OPERATOR PROBLEM

- CEM operator receives offers $\beta_1, \beta_2, \ldots, \beta_M$ from which it determines the optimum schedule.

$$\sum^{opt} = \left\{ u^{opt}, p^{opt}, r^{opt} \right\}$$

that minimizes the total costs $P(D,R)$ incurred by the CEM operator.

- The CEM's costs for its purchases from unit $i$ are

$$F_i(u_i, p_i, r_i) = \sum_{t=1}^{T} \left[ b_i^F(p_{it}) + b_i^S(\tau_{it-1})(1 - u_{it-1}) \right] u_{it}$$

with
THE \textit{CEM} OPERATOR PROBLEM

$$\tau_{i,t} = \left( \tau_{i,t-1} + \Delta \right) \left( 1 - u_{i,t} \right), \quad t = 1, 2, \ldots, T$$

and $\tau_{i,0}$ specified

\begin{itemize}
  \item The total \textit{CEM} costs are
    \begin{align*}
    \sum_{i=1}^{M} F_i \left( u_i, p_i, r_i \right) &= \sum_{i=1}^{M} \sum_{t=1}^{T} \left\{ b_i^F \left( p_{i,t} \right) + b_i^S \left( \tau_{i,t-1} \right) \left( 1 - u_{i,t-1} \right) \right\} u_{i,t}
    \end{align*}
\end{itemize}

\begin{itemize}
  \item We next state the \textit{CEM} operator problem
\end{itemize}
THE CEM OPERATOR PROBLEM STATEMENT

\[ P(D, R) = \min_{u, p, r} \left\{ \sum_{i=1}^{M} \sum_{t=1}^{T} \left[ b_i^F(p_{i,t}) + b_i^S(\tau_{i,t-1})(1 - u_{i,t-1}) \right] u_{i,t} \right\} \quad (a) \]

s.t.

\[ D_t - \sum_{i=1}^{M} p_{i,t} u_{i,t} = 0 \quad \forall t = 1, 2, \ldots, T \quad (b) \]

\[ R_t - \sum_{i=1}^{M} r_{i,t} u_{i,t} \leq 0 \]
THE CEM OPERATOR PROBLEM
STATEMENT

\[ p_{i \text{min}} \leq p_{i \text{it}} \leq p_{i \text{max}} \]

\[ 0 \leq p_{i \text{it}} \leq a_{i \text{it}} \]

\[ 0 \leq r_{i \text{it}} \leq \min \{r_{i \text{max}}, p_{i \text{max}} - p_{i \text{it}}\} \]

\[ u_{i \text{it}} \in \{0,1\} \]

\[ \tau_{i,t} \text{ satisfies the } T_{i}^{d} \text{ limits} \]

\[ \tau_{i,0} \text{ is given} \]
THE *CEM* OPERATOR PROBLEM STATEMENT

We define the set of feasible operating schedules for unit $i$ to be

$$\Omega_i \left( a_i \right) \triangleq \left\{ \Sigma_i : \Sigma_i = \left\{ u_i, p_i, r_i \right\} \text{satisfies (c)} \right\}$$

The problem is restated as $(P)$:
THE **CEM** OPERATOR PROBLEM

**STATEMENT**

\[
P(D,R) = \min_{u,p,r} \left\{ \sum_{i=1}^{M} \sum_{t=1}^{T} \left[ b_i^F(p_{it}) + b_i^S(\tau_{it-1})(1-u_{it-1}) \right] u_{it} \right\}
\]

**s.t.**

\[
D_t - \sum_{i=1}^{M} p_{it} u_{it} = 0 \quad \forall t = 1,2, \ldots, T \\
R_t - \sum_{i=1}^{M} r_{it} u_{it} \leq 0
\]

\[
\{u_i, p_i, r_i\} \in \Omega_i(a_i) \quad \forall i = 1,2, \ldots, M
\]
CHARACTERISTICS OF \( (P) \)

- The objective function is nonconvex
- The state space admits \( T^u_i \) and \( T^d_i \) constraints and is discrete in \( u_i \) so that the set of feasible schedules \( \Omega_i(a_i) \) is nonconvex
- For the day-ahead markets, \( T = 24 \text{ hours} \); the EWPP uses \( T = 24 \text{ hours} \) but the basic period is half an hour and so the optimization involves 48 periods over the \( T = 24 \text{ hours} \) scheduling horizon
THE CEM OPERATOR PROBLEM

- The CEM operator therefore solves a transmission unconstrained UC problem over the scheduling horizon.
- This results in a large-scale nonlinear optimization program with both continuous and discrete decision variables.
- The optimal values determine the prices and purchase/sale quantities for each period in the scheduling horizon.
THE CEM OPERATOR PROBLEM

\[
\min \sum \left[ \text{variable charges} + \text{startup charges} \right]
\]

s.t.

- meeting system demand requirements
- meeting system reserves requirements
- minimum and maximum output limits
- offer availability limits
- reserves contribution limits
- start-up/shut-down constraints
- on/off status

for each offer
THE CEM FRAMEWORK

- System schedule
  - Output MW
  - Time

- Prices
  - System marginal price
  - System reserves price
  - Time

- Load and reserves requirements forecasts

- Seller 1
  - \( \beta_1 \)

- Seller M
  - \( \beta_M \)
THE CEM OPERATOR PROBLEM SOLUTION

- For each unit $i$, the solution yields
  - the optimal operations schedule – the vector of each period’s generation output and reserves contribution
  - the set of optimal marginal prices for each period

- The optimal system schedule is then constructed from the set of optimal unit schedules
LAGRANGIAN RELAXATION

We define the $T$– dimensional vectors

$$\lambda = \begin{bmatrix} \lambda_1, \lambda_2, \ldots, \lambda_T \end{bmatrix}^T$$

and

$$\mu = \begin{bmatrix} \mu_1, \mu_2, \ldots, \mu_T \end{bmatrix}^T \geq \begin{bmatrix} 0 \end{bmatrix}^T$$

For the Lagrangian relaxation of the CEM operator problem, we construct the Lagrangian
LAGRANGIAN RELAXATION

\[
\min_{u, p, r} \left\{ \sum_{i=1}^{M} \sum_{t=1}^{T} \left[ b_i^F (p_{i,t}) + b_i^S (\tau_{i,t-1})(1 - u_{i,t-1}) \right] u_{i,t} + \sum_{t=1}^{T} \lambda_t \left( D_t - \sum_{i=1}^{M} p_{i,t} u_{i,t} \right) + \sum_{t=1}^{T} \mu_t \left( R_t - \sum_{i=1}^{M} r_{i,t} u_{i,t} \right) \right\}
\]

- The constraints require that

\[\left\{ u_i, p_i, r_i \right\} \in \Omega_i \left( a_i \right) \quad \forall i = 1, 2, \ldots, M\]
LAGRANGIAN RELAXATION

- We can restate the Lagrangian relaxation as

\[
\min_{u,p,r} \left\{ \sum_{i=1}^{M} \sum_{t=1}^{T} \left[ b^F_i(p_{i,t}) + b^S_i(\tau_{i,t-1})(1-u_{i,t-1}) - \lambda_ip_{i,t} - \mu_ir_{i,t} \right] \right. \\
\left. \quad u_{i,t} + [\lambda^T D + \mu^T R] \right\}
\]

s.t.

\[\{u_i, p_i, r_i\} \in \Omega_i(a_i) \quad \forall i = 1, 2, \ldots, M\]

- As \(\lambda^T D\) and \(\mu^T R\) are constant for given \(\lambda\) and \(\mu\), they do not affect the minimization and so we remove them from the minimand.
We define the *modified Lagrangian relaxation problem*

\[ \phi(\lambda, \mu; D, R) \triangleq \min_{u_i, p_i, r_i} \left\{ \sum_{i=1}^{M} \sum_{t=1}^{T} \left[ b_i^F(p_{i,t}) + b_i^S(\tau_{i,t-1})(1 - u_{i,t-1}) - \lambda_t p_{i,t} - \mu_t r_{i,t} \right] u_{i,t} \right\} \]

s.t.

\[ \{u_i, p_i, r_i\} \in \Omega_i(a_i) \quad \forall i = 1, 2, \ldots, M \]
LAGRANGIAN RELAXATION

- Since there is no inter-unit coupling in the constraints, the $\phi$ function is an additively separable sum with each seller contributing terms that depend only on that seller; such structure allows the decomposition of the problem into $M$ subproblems.

- We decompose the $LR$ formulation into $M$ decoupled subproblems which we solve individually.
THE SELLER \( i \) SUBPROBLEM

- The subproblem for seller \( i \) is

\[
\phi_i(\lambda, \mu) = \min_{u_i, p_i, r_i} \left\{ \sum_{t=1}^{T} \left[ b_i^F(p_{i,t}) + b_i^S(\tau_{i,t-1}) \right] \right. \\
\left. (1 - u_{i,t-1}) - \lambda_t p_{i,t} - \mu_t r_{i,t} \right\} u_{i,t}
\]

s.t.

\[
\left\{ u_i, p_i, r_i \right\} \in \Omega_i\left(a_i\right) \quad \forall i = 1, 2, \ldots, M
\]
This subproblem is somewhat easier to solve even with the minimum up and down time constraints.

Given $\lambda_t$ and $\mu_t$, whenever the unit $i$ is operating, i.e., $u_{i,t} = 1$, we can determine the optimal $p_{i,t}$ and $r_{i,t}$ that minimize the time subperiod $t$ CEM costs.
THE SELLER $i$ SUBPROBLEM

- It remains to determine in which periods unit $i$ is on, taking into account the start-up price and the specified \textit{minimum up – and down – time constraints}; \textit{dynamic programming} provides an effective solution approach to this problem.

- For given $\lambda$ and $\mu$, each of the $M$ subproblems may be solved separately resulting in \textit{numerical efficiency}.
For specified $D$ and $R$, $\phi(\lambda, \mu; D, R)$ is a concave function of $\lambda$ and $\mu$ and

$$P(D, R) \geq \phi(\lambda, \mu; D, R) \quad \forall \lambda, \forall \mu \geq 0$$

We make use of these properties to find a near-optimal solution to the primal CEM operator problem $(P)$ by maximizing $\phi(\lambda, \mu; D, R)$.
SOLUTION OF THE DUAL

We define

\[ L(D, R) \triangleq \max \left\{ \phi(\lambda, \mu; D, R) : \lambda, \mu \geq 0 \right\} \]

and denote by \((\lambda^*, \mu^*)\) the optimal Lagrange multipliers that maximize \(\phi\), i.e.,

\[ L(D, R) = \phi(\lambda^*, \mu^*; D, R) \]

Clearly,

\[ P(D, R) \geq L(D, R) \]
SOLUTION OF THE DUAL

so that the maximization of \( \phi(\lambda, \mu; D, R) \)

provides a tight lower bound on the optimal cost \( P(D, R) \) of \( (P) \)

Since \( \phi(\lambda, \mu; D, R) \) is concave in \( \lambda \) and \( \mu \), any

local maximum is a global maximum so that the

maximization of \( \phi \) is attractive
SOLUTION OF THE DUAL

- As a by-product of maximizing \( \phi(\lambda, \mu; D, R) \), we obtain the optimal Lagrange multipliers \( \lambda^* \) and \( \mu^* \) and the system schedule \( \Sigma^* = \{u^*, p^*, r^*\} \) by solving the Lagrangian relaxation problem for \( \lambda = \lambda^* \) and \( \mu = \mu^* \).

- The schedule \( \Sigma^* = \{u^*, p^*, r^*\} \) must satisfy the feasibility constraints

\[
\Sigma^*_i = \{u^*_i, p^*_i, r^*_i\} \in \Omega_i(a_i), \ i = 1, \ldots, M
\]
SOLUTION OF THE DUAL

- In certain cases, $\sum^*$ satisfies the demand and reserves constraints making it feasible for $(P)$; if, in addition, $\sum^*$ satisfies the complementary slackness condition, $\sum^*$ is, in fact, the optimal schedule to the primal problem, i.e., $\sum^* = \sum^{opt}$

- In general, however, $\sum^*$ is infeasible for $(P)$ and therefore cannot be optimal
SOLUTION OF THE DUAL

- $\sum^*$ may be modified to obtain a *near-optimal* schedule $\hat{\sum} = \{\hat{u}, \hat{p}, \hat{r}\}$ which is feasible for $(P)$ using a supergradient algorithm for the maximization of $\phi(\lambda, \mu; D, R)$ to determine $\lambda^*$ and $\mu^*$ and the feasible schedule $\hat{\sum}$.

- A measure of the closeness of $\hat{\sum}$ to the optimal system schedule $\sum^{opt} = \{u^{opt}, p^{opt}, r^{opt}\}$ is
SOLUTION OF THE DUAL

\[
\sum_{i=1}^{M} F_i \left( \hat{u}_i, \hat{p}_i, \hat{r}_i \right) - L(D, R) \left/ L(D, R) \right.
\]

which provides an overestimate of the fraction of the optimal cost achieved using the schedule

\[
\hat{\sum} = \{ \hat{u}, \hat{p}, \hat{r} \}
\]

This discussion provides the basis for the LR approach
The optimal system schedule $\left\{ u_{\text{opt}}, p_{\text{opt}}, r_{\text{opt}} \right\}$ is difficult to find; the schedule $\hat{\Sigma} = \left\{ \hat{u}, \hat{p}, \hat{r} \right\}$ is close to the optimal system schedule, by virtue of the above discussion.

We assume that the difference between the operating schedule $\left\{ \hat{u}, \hat{p}, \hat{r} \right\}$ and $\left\{ u_{\text{opt}}, p_{\text{opt}}, r_{\text{opt}} \right\}$ is negligibly small.
PROPERTIES OF $L(D, R)$

- $L(D, R)$ is a convex function of $D$ and $R$; this property is derived by considering the nature of the CEM operator problem.

- Let $\left(\lambda^*, \mu^* \right)$ be the optimal Lagrange multipliers that maximize the Lagrangian $\phi(\lambda, \mu; D, R)$, i.e.,

$$\phi(\lambda^*, \mu^*; D, R) = L(D, R)$$
The vector \( \begin{bmatrix} \lambda^* T \\ \mu^* T \end{bmatrix} \) is a supergradient to \( L(\cdot, \cdot) \) at \((D, R)\), since \( \forall \Delta D, \forall \Delta R \)

\[
L(D + \Delta D, R + \Delta R) \geq L(D, R) + \lambda^* \Delta D + \mu^* \Delta R
\]

and \( \begin{bmatrix} \lambda^* T \\ \mu^* T \end{bmatrix} \) is the gradient of \( L(\cdot, \cdot) \) whenever \( L(\cdot, \cdot) \) is differentiable
KEY ASSUMPTION

We assume that the optimal Lagrange multiplier

\[ \lambda^* \left( \mu^* \right) \]

associated with the demand (system reserves requirements) in period \( t \) differs negligibly from the system marginal (reserves) price

\[ \lambda^{opt} \left( \mu^{opt} \right) \] in period \( t \)
THE OPTIMAL OFFER STRATEGY PROBLEM

- The problem of seller $i$ is to specify his strategy in terms of the submitted offer $\beta_i$ so as to maximize his profits.
- The bids $\beta_j, j = 1, 2, ..., i - 1, i + 1, ..., M$ of the other competing sellers are fixed but unknown to seller $i$.
- The CEM operator determines the optimal system prices $\left(\lambda^*, \mu^*\right)$, the operating schedules of seller $i$. 
THE OPTIMAL OFFER STRATEGY PROBLEM

\[ \Sigma_i^* = \{ u_i^*, p_i^*, r_i^* \} \] and constructs the optimal system schedule \( \Sigma^* = \{ u^*, p^*, r^* \} \)

Clearly, \( \lambda^* (\mu^*) \) depends on all the offers \( \beta_j \)

\( j = 1, 2, \ldots, M \); however, seller \( i \) has control over only \( \beta_i \) and so from the seller \( i \) point of view

\[ \lambda^* = \lambda^* (\beta_i) \text{ and } \mu^* = \mu^* (\beta_i) \]

and \( \Sigma_i^* \) depends consequently on \( \beta_i \)
The *LR* approach determines \( \Sigma_i^* = \{ u_i^*, p_i^*, r_i^* \} \)

that satisfies

\[
\min_{u_i, p_i, r_i} \left\{ \sum_{t=1}^{T} \left[ b^F_i \left( p_{i,t} \right) + b^S_i \left( \tau_{i,t-1} \right) \left( 1 - u_{i,t-1} \right) - \lambda_t^* (\beta_i) p_{i,t} - \mu_t^* (\beta_i) r_{i,t} \right] u_{i,t} : \{ u_i, p_i, r_i \} \in \Omega_i (a_i) \right\}
\]
The generation costs incurred in each time period \( t \) is the sum of the variable costs

\[
[ c_i^F (p_{it}^*) + c_i^S (\tau_{it-1})(1-u_{it-1}^*) ] u_{it}^*
\]

so that the total costs of unit \( i \) for the \( T \) periods are

\[
\sum_{t=1}^{T} [ c_i^F (p_{it}^*) + c_i^S (\tau_{it-1})(1-u_{it-1}^*) ] u_{it}^*
\]
THE PROFITS OF SELLER $i$

- The price paid to generator $i$ in each time period $t$ is $\lambda^*_i(\beta_i)$ per MWh of energy and $\mu^*_i(\beta_i)$ per MW of reserves provided.

- The profits $\Pi_i(\beta_i; \lambda^*_i(\beta_i), \mu^*_i(\beta_i))$ of seller $i$ are equal to the revenues less the costs incurred:

$$\Pi_i(\beta_i; \lambda^*_i(\beta_i), \mu^*_i(\beta_i)) = \sum_{t=1}^{T} \left[ \lambda^*_i(\beta_i) p_{i,t} + \mu^*_i(\beta_i) r_{i,t} - c^F_i(p^*_{i,t}) + c^S_i(\tau^*_{i,t-1})(1-u^*_{i,t-1}) \right] u^*_{i,t}$$
THE PROFITS OF SELLER \( i \)

- We define the loss function

\[
\Lambda_i \left( \beta_i ; \lambda_i^* (\beta_i), \mu_i^* (\beta_i) \right) \triangleq - \Pi_i \left( \beta_i ; \lambda_i^* (\beta_i), \mu_i^* (\beta_i) \right)
\]

- The optimal offer strategy problem whose objective is to maximize \( \Pi_i \left( \beta_i ; \lambda_i^* (\beta_i), \mu_i^* (\beta_i) \right) \) is identical to that selected to minimize the losses \( \Lambda_i \left( \beta_i ; \lambda_i (\beta_i), \mu_i (\beta_i) \right) \) since

\[
\max_{\beta_i} \left\{ \Pi_i \left( \beta_i ; \lambda_i^* (\beta_i), \mu_i^* (\beta_i) \right) \right\} = - \min_{\beta_i} \left\{ \Lambda_i \left( \beta_i ; \lambda_i^* (\beta_i), \mu_i^* (\beta_i) \right) \right\}
\]
THE OPTIMAL OFFER STRATEGY PROBLEM

\[
\begin{align*}
\min_{b_i^F(\cdot), b_i^S(\cdot), a_i \geq 0} \left\{ \sum_{t=1}^{T} \left[ c_i^F(p_{i,t}^*) + c_i^S(\tau_{i,t-1}^*) \left(1 - u_{i,t-1}^*\right) - \lambda_t^*(\beta_i)p_{i,t}^* - \mu_t^*(\beta_i)r_{i,t}^* \right] u_{i,t}^* \right\}
\end{align*}
\]

\{u_{i,t}^*, p_{i,t}^*, r_{i,t}^*\} minimizes the problem

\[
\begin{align*}
\min_{\underline{u}_i, \underline{p}_i, \underline{r}_i} \left\{ \sum_{t=1}^{T} \left[ b_i^F(p_{i,t}) + b_i^S(\tau_{i,t-1}) \left(1 - \underline{u}_{i,t-1}\right) - \lambda_t^*(\beta_i)p_{i,t} - \mu_t^*(\beta_i)r_{i,t} \right] \underline{u}_{i,t} \right\}
\end{align*}
\]
THE OPTIMAL OFFER STRATEGY PROBLEM

\[ \min \max_{i,t} \]

\[ p_{i,t}^{\min} \leq p_{i,t} \leq p_{i,t}^{\max} \]

\[ 0 \leq p_{i,t} \leq a_{i,t} \]

\[ 0 \leq r_{i,t} \leq \min \{r_{i,t}^{\max}, p_{i,t}^{\max} - p_{i,t}\} \quad \forall t = 1, 2, ..., T \]

\[ u_{i,t} \in \{0, 1\} \]

\[ \tau_{i,t} \text{ satisfies the } T_i^d \text{ constraints} \]

\[ \tau_{i,0} \text{ is given} \]
EVERY SELLER IS A PRICE TAKER

- **Basic assumption**: the offer of any seller has a negligible effect on the system marginal and reserves prices; it follows that \( \lambda^*(\beta_i) = \lambda^0 \) and \( \mu^*(\beta_i) = \mu^0 \)

- Such a situation can be considered to exist when no single seller possesses a large fraction of the total capacity or MWh generation in the CEM.
EVERY SELLER IS A PRICE TAKER

- The offer of any seller has a negligible effect on the system marginal and reserves prices.
- In essence, the assumption states that the market price is determined by the offers of the set of competing sellers; the CEM is perfectly or purely competitive from the point of view of generator $i$, i.e., the market clearing prices are independent of the offer and hence every generator is a price taker.
In terms of the unit $i$ profits $\prod_i (\beta_i ; \lambda^0, \mu^0)$ (losses $\Lambda_i (\beta_i ; \lambda^0, \mu^0)$), the optimal offer strategy is

$$\min_{b_i^F(\cdot), b_i^S(\cdot), a_i \geq 0} \left\{ \sum_{t=1}^{T} \left[ c_i^F (p_{i,t}^*) + c_i^S (\tau_{i,t-1})(1 - u_{i,t-1}^*) \right] - \lambda_t^0 \left[ p_{i,t}^* - \mu_t^0 r_{i,t}^* \right] u_{i,t}^* \right\}$$

where $\{u_i^*, p_i^*, r_i^*\}$ minimizes the problem.
PERFECT COMPETITION

\[
\min_{u_i, p_i, r_i} \left\{ \sum_{t=1}^{T} \left[ b_i^F (p_{i,t}) + b_i^S (\tau_{i,t-1})(1 - u_{i,t-1}) - \lambda_t^0 p_{i,t} \right. \right. \\
\left. \left. - \mu_t^0 r_{i,t} \right] u_{i,t} : \left\{ u_i, p_i, r_i \right\} \in \Omega_i (a_i) \right\} \quad (ii)
\]

The structural similarity between the minimizations in (i) and (ii) allows the use of the following lemma
Lemma: a global optimal solution to the problem in (i) and (ii) is the offers

\[ \beta_{opt}^{i} = \left\{ b_{i}^{F}(\cdot), b_{i}^{S}(\cdot), a_{i} \right\}, \]

where

\[ b_{i}^{F}(p) = c_{i}^{F}(p) \quad \forall p \in \left[ p_{i}^{\text{min}}, p_{i}^{\text{max}} \right] \]

\[ b_{i}^{S}(\tau) = c_{i}^{S}(\tau) \quad \forall \tau \leq 0 \]

\[ a_{i,t} = p_{i}^{\text{max}} \quad \forall t = 1, 2, \ldots, T \]
PROPERTIES OF THE OPTIMAL OFFER STRATEGY

- $\beta_{i}^{opt} = \left\{ c_{i}^{F}(\cdot), c_{i}^{S}(\cdot), p_{i}^{max} \right\}$ is a globally optimal offer strategy and therefore, no other offer strategy can result in greater profits to the seller.

- The global optimality is independent of the system price pair $(\lambda^{0}, \mu^{0})$ and so regardless of the prices that may be realized during the schedule horizon, the offer is optimal if it equals $\beta_{i}^{opt}$. 
A seller $i$ whose unit has a large $T_i^u$ may feel it is to his advantage to *under offer* his costs to compensate for the fact that the constraints on the unit will hinder its ability to get scheduled; such a strategy abandons a known global optimum for another that may not be so
PROPERTIES OF THE OPTIMAL OFFER STRATEGY

- Analogously, a generator with a short $T_i$ may be inclined to over offer, believing that he can cash in on the unit’s higher level of commitment; again, this strategy is ineffective for the identical reasons.

- A salient feature of this optimal offer strategy is that it reveals the true costs of operation of the unit to the CEM dispatcher; this highly desirable
outcomes is due to the construction of this auction for the right to serve load in the CEM since every accepted offer from a supplier is paid, at least, its offered price.

We refer to the optimal offer as a truth-revealing offer; such offers are highly useful in price discovery.
The optimal offer has the components:

- Offer variable price = variable costs
- Offer start-up price = start-up costs
- Offer capacity = maximum capacity in each subperiod

This strategy is termed offering at cost.
ADDITIONAL OPTIMAL STRATEGY PROPERTIES

- Provides analytical estimates of the
  - scheduling horizon expected seller profits
  - profit volatility to measure risk

- Allows evaluation of “what if” situations; e.g.,
  sensitivity analysis of changes in profits with respect to an investment to improve unit efficiency
THE SELLER PERSPECTIVE

profits = payments - costs

seller 1

seller i

seller M

variable costs

start-up costs

\[
\beta_i
\]

\[\frac{S}{h} \text{ MW} \]

\[s \text{ time} \]

variable price

offered capacity

start-up price

\[\beta_M\]

CEM operator

system schedule and prices

system marginal price

reserves price

output MW

unit 1

time

output MW

unit 2

time

\[\cdot\cdot\cdot\]

output MW

unit M

time

\[\cdot\cdot\cdot\]
OFFER UNIT COST DATA

\[ \text{\£/h} \]

\[ 1600 \]

\[ 1200 \]

\[ 800 \]

\[ 400 \]

\[ 0 \]

\[ 100 \]

\[ 200 \]

\[ 300 \]

\[ 400 \]

\[ 500 \]

\[ 600 \]

\[ 700 \]

\[ \text{output power (MW)} \]

\[ m^1 \]

\[ m^2 \]

\[ m^3 \]

\[ e^1 \]

\[ e^2 \]
OFFER PARAMETERS

price to the CEM in £/h

offer price

$\eta^1$

$\eta^2$

$\eta^3$

$\varepsilon^1$

$\varepsilon^2$

$p_{\text{min}}$

$p_{\text{max}}$

$b^0_i$
PROFITS vs. WILLANS LINE
PARAMETER $\eta^1$

profits ($10^5 £/week$)
PROFITS vs. WILLANS LINE
PARAMETER $\eta^2$

$profits \ (10^5 \ £/week)$
KEY POINTS

- We introduced the very general and comprehensive CEM framework.
- The analytical model explicitly accounts for the sealed bid auction mechanism and the constraints associated with the generation of electrical power in the development of the optimal offer strategy.
- The analytical tools provide sellers with valuable information concerning performance of units in a competitive market.