14. Introduction to Unit Commitment

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TOPICS

- The unit commitment (UC) problem statement
- Thermal system description
- UC problem formulation and statement
- Solution approaches
- Issues in UC
UC DECISION PROCESS

load forecast

desired spinning reserves

unit status and characteristics

unit commitment

commitment schedule
MATCHING SUPPLY AND DEMAND

MW

total capacity of the available
generation units

committed capacity schedule

forecasted thermal system load
UC ROLE IN SCHEDULING

- Energy storage scheduling
- Nuclear refueling and production scheduling
- Maintenance scheduling
- Hydro scheduling

- Load forecasting
- Unit commitment
- Reliability analysis
- Transaction evaluation

- Economic dispatch
THERMAL SYSTEM UNIT TYPES

- Base – loaded units
  - are continuously on line
  - include, typically, nuclear, geothermal, very large steam units and also all must – run units

- Cycling units
  - have on/off characteristics with minimum
THERMAL SYSTEM UNIT TYPES

up/down times specified

- are, typically, steam units

- Peaking units

  - have on/off and are, typically, loaded at full capacity

  - good examples are combustion turbines
THERMAL SYSTEM DESCRIPTION

- Constraint types considered
  - unit
  - plant related
  - system wide

- Cost components of scheduling unit operations
  - fuel
  - maintenance
  - start – up/shutdown
THE UNIT COMMITMENT TASK

- **Scope**: to determine the minimum cost strategies for the start – up and shutdown of thermal units to supply the forecasted thermal load for a given period in a manner consistent with the generation equipment limitations and operating policies
THE UNIT COMMITMENT TASK

- **Period**: typically, from one day to one week
- **Basic unit of time**: typically, one hour
- **Decisions**: the schedule of the hourly start – up and shutdown of thermal units
- **By – products**: hourly generation level for each thermal unit
Minimum output

Maximum output

Minimum up time

Minimum down time

Start-up delay

Derating

Ramp rate (load pick up/reduction per minute)
REPRESENTATIVE PLANT RELATED CONSTRAINTS

- Maximum number of units which can be started up in a period $k$ taking into account:
  - crew limitations
  - auxiliary system constraints

- Maximum output from a plant

- Fuel constraints
REPRESENTATIVE SYSTEM-WIDE CONSTRAINTS

- **Load supply**: commit sufficient generation to supply the forecasted load

- **Spinning reserves**: commit adequate number of generation units to meet the spinning reserves requirements

- **Operating reserves**: commit adequate number of generation units to meet the operating reserves requirements
REPRESENTATIVE SYSTEM-WIDE CONSTRAINTS

- **Area Protection**: satisfy area load protection requirements
- **Environmental**: satisfy emissions and effluent discharge limitations
- **Transmission**: comply with tie line flow limitations
- **Fuel**: comply with storage and inventory restrictions
CONTRIBUTORS TO RESERVES

- Synchronized units that operate below full capacity
- Gas turbine units that can be synchronized in $t$ minutes or less
- Interruptible loads (e.g., pumping loads)
- Purchase capacity interchange contracts (limited by tie-line constraints)
RESERVES REQUIREMENTS

- Security and reliability considerations impose reserves requirements for every system.

- Reserves requirements must be aligned with the level of security and reliability that the system wishes to maintain: the higher the reliability, the higher the reserves requirements for a given system.
RESERVES REQUIREMENTS

- Reserves requirements are typically expressed as a fraction of the peak load for a given period: in essence, they constitute a deterministic criterion and act as a proxy for the probabilistic reliability measures.

- The reserves requirements are a function of:
  - system net load
RESERVES REQUIREMENTS

- capacity obligations to other entities
- capacity of largest unit committed
- generation of the most heavily loaded unit
- level of interruptible imports and loads

Reserves provide critically important insurance for the system operator
RESERVES DEFINITIONS

- **t – minute reserves of unit i**: the additional load that unit i is capable to pick up in t minutes, with respect to its current operating point

  \[ r^t_i(k) = \min \{ p^{\text{max}}_i - p_i(k), t \cdot (\text{ramp rate of unit i}) \} \]

- **t – minute system reserves**: the additional load which the system is capable of picking up in t minutes from its current state

  \[ \text{period k system reserves} = \sum_{\text{commited units i}} r^t_i(k) \]
RESERVES DEFINITIONS

- Reserves are characterized by the:
  - response time, and
  - on-line/off-line status of contributors

- Reserves depend on the current system loading and on the loading of the committed generating resources

- *Spinning reserves*: usually defined as the 5 – minute system reserves provided solely by on-line units
RESERVES DEFINITIONS

- **Operating reserves**: usually defined as the 10–minute or the 30–minute system reserves provided solely by the on-line units.

- **Supplemental or backup reserves**: capacity that may become available on a longer basis, typically one hour, provided by units which need not be already synchronized.
The generator economics are, usually, summarized in terms of the input–output curves. The input–output curve determines the generator costs of production in units of $/h and the marginal costs of production in units of $/MWh. These curves are idealizations of the input–output characteristics of a unit.
THERMAL UNIT INPUT – OUTPUT CURVE

input in $10^6 \text{ Btu/h}$

minimum capacity

maximum capacity

output in MW
THERMAL UNIT INPUT – OUTPUT CURVE

heat rate = \frac{\text{input}}{\text{output}}

incremental heat rate = \frac{\text{incremental input}}{\text{incremental output}}

minimum capacity

maximum capacity

a possible operating point

input in $10^6$ Btu/h

output in MW
HEAT RATE CHARACTERISTICS

heat rate in $10^6$ Btu/MWh

minimum capacity
rated capacity
maximum capacity

output in MW
VALVE – POINT LOADING

- First valve: Fully open
- Second valve
- Third valve
- Fourth valve
- Fifth valve

Input in $10^6$ Btu/h vs. Output in MW

Actual thermal unit input-output curve
Idealized input-output curve
INCREMENTAL CHARACTERISTICS

INCREMENTAL HEAT RATE

10^6 Btu/MWh

MINIMUM
CAPACITY

MAXIMUM
CAPACITY

OUTPUT
IN MW
**INPUT – OUTPUT CURVE**

**Graph Description**:
- The graph illustrates the relationship between input and output for a system.
- The x-axis represents the output in MW.
- The y-axis represents the input in $10^6$ Btu/h.
- The graph shows two distinct regions:
  - **Minimum Capacity**: The point where input is at its minimum.
  - **Maximum Capacity**: The point where output is at its maximum.
- The heat rate, defined as $\Delta h / \Delta p$, is shown in the diagram.
- The inc heat rate is calculated as $\frac{\Delta h}{\Delta p}$.
- The input output is also highlighted.

Mathematical Equation:
\[
\text{inc heat rate} \quad \frac{\Delta h}{\Delta p} = \text{inc input output}
\]
VALVE – POINT LOADING

The diagram illustrates a typical actual thermal unit incremental heat rate curve and an idealized incremental heat rate curve. The x-axis represents output in MW, and the y-axis represents incremental heat rate in $10^6$ Btu/MWh.

Key points on the graph include:

1. Idealized incremental heat rate curve
2. Typical actual thermal unit incremental heat rate curve
3. Points 1, 2, 3, 4, 5 on the graph
VALVE – POINT LOADING

- Idealized average heat rate curve
- Incremental heat rate curve
- Actual average heat rate curve
- Point of maximum efficiency
- Valve points

Output in MW

- 10^6 Btu/MWh

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NOTATION

\[ i \quad = \quad \text{unit index with values 1,2, \ldots ,N} \]

\[ N \quad = \quad \text{number of thermal units} \]

\[ k \quad = \quad \text{time period with values 1,2, \ldots ,K} \]

\[ K \quad = \quad \text{study numbered time periods in study period} \]

\[ u_i(k) \quad = \quad \text{status of unit } i \text{ in period } k \]
NOTATION

\[ u_i(k) = \begin{cases} 
  m > 0 & \text{unit } i \text{ is up in period } k \text{ and also in the previous } (m - 1) \text{ periods} \\
  \ell < 0 & \text{unit } i \text{ is down in period } k \text{ and also in the previous } - (\ell + 1) \text{ periods} 
\end{cases} \]

\[ p_i(k) = \text{output level in MW of unit } i \text{ in period } k \]
NOTATION

\[ u_i = \begin{bmatrix} u_i(1), u_i(2), \ldots, u_i(K) \end{bmatrix}^T \in \mathbb{R}^K \]

\[ p_i = \begin{bmatrix} p_i(1), p_i(2), \ldots, p_i(K) \end{bmatrix}^T \in \mathbb{R}^K \]

\[ u = \begin{bmatrix} u_1 \mid u_2 \mid \ldots \mid u_N \end{bmatrix} \in \mathbb{R}^{K \times N} \]

\[ p = \begin{bmatrix} p_1 \mid p_2 \mid \ldots \mid p_N \end{bmatrix} \in \mathbb{R}^{K \times N} \]
NOTATION

\[
T_{i}^{u} = \text{minimum up time for unit } i
\]

\[
T_{i}^{d} = \text{minimum down time for unit } i
\]

\[
\tau_{i} = \text{cooling down time constant for unit } i
\]

\[
d(k) = \text{demand requirements in period } k
\]

\[
\beta(k) = \text{reserve requirements in period } k
\]
NOTATION

\[ r^t_i = r_i[u_i(k), p_i(k)] \]

= \text{t – minute reserves for unit } i \text{ in period } k

\[ S_i = \text{set of all feasible schedules for unit } i \]

for the study period

\[ S = \left\{ (u, p) : (u_i, p_i) \in S_i \right\} \]
UNIT FUEL COSTS

fuel costs in $/h

\[
f_i^F \left[ u_i(k), p_i(k) \right] = \begin{cases} 
w_i \left[ p_i(k) \right] & \text{if } u_i(k) > 0 \\
0 & \text{if } u_i(k) \leq 0
\end{cases}
\]

no-load generation cost
curve derived from the input-output curve

\( w_i(p_i) \)

\( f_i^F \left[ u_i(k), p_i(k) \right] \)

\( p_i^{\text{min}} \)

\( p_i^{\text{max}} \)

\( p_i \)

\( MW \)
UNIT MAINTENANCE COSTS

\[ f_i^M [u_i(k), p_i(k)] = \begin{cases} 
  m_i^A + m_i^B p_i(k) & \text{if } u_i(k) > 0 \\
  m_i^A & \text{if } u_i(k) \leq 0 
\end{cases} \]
UNIT START–UP COSTS

\[ f_i^S[u_i(k), p_i(k)] = f_i^S[u_i(k)] \]

\[ = \begin{cases} 
  s_i^A + s_i^B \left[ 1 - e^{\frac{u_i(k-1)}{\tau}} \right] & \text{if } u_i(k) = 1 \\
  0 & \text{otherwise} 
\end{cases} \]
UNIT START – UP COSTS

startup costs

costs of starting turbine alone

costs of cooling rate

costs of cold start

$\sum_{i}^{A} + \sum_{i}^{B}$

$\sum_{i}^{A}$
The aim is to minimize the total costs incurred in starting up and shutting down the generating system.

Total costs are the sum of the fuel, maintenance and start up costs.

For each unit $i$, we define
The objective function is formulated as

\[ f_i[u_i(k), p_i(k)] \triangleq f_i^F[u_i(k), p_i(k)] + f_i^M[u_i(k), p_i(k)] \]

\[ + f_i^S[u_i(k)] \]

The objective function is formulated as

\[ f(u, p) = \sum_{k=1}^{K} \sum_{i=1}^{N} f_i[u_i(k), p_i(k)] \]
**UC PROBLEM FORMULATION**

\[ \min \quad f(u, p) = \sum_{k=1}^{K} \sum_{i=1}^{N} f_i \left[ u_i(k), p_i(k) \right] \]

s.t.

\[ \sum_{i=1}^{N} p_i(k) = d(k) \quad k = 1, 2, \ldots, K \]

\[ \sum_{i=1}^{N} r_i \left[ u_i(k), p_i(k) \right] \geq \beta(k) \]

\[ (u, p) \in S \]
UC PROBLEM CHARACTERISTICS

- Large – scale problem due to the many periods and the system size
- Mixed – integer nonlinear programming problem characterized by
  - integer commitment decision variables \( u_i(k) \)
  - nonlinear and non-continuous objective function
  - nonlinear and non-continuous functions
  - a highly constrained region \( S \)
A salient characteristic is that the cost function is additively separable:

\[ f(u, p) = \sum_{i=1}^{N} \left\{ \sum_{k=1}^{K} f_i[u_i(k), p_i(k)] \right\} \]

The effective solution must take advantage of the problem characteristics and, in general, requires heuristics for computational tractability.