University of Illinois **Fall 2016**

ECE 562: Problem Set 5
Coding and Decoding, Equalization

Due: Thursday November 3 in class
Reading: Lecture Notes 13-16; Chapter 5 and Section 7.1 of Madhow

1. [**Golay Code with BPSK on an AWGN Channel**]
The *Golay* code is a (23, 12) linear code with $d_{\min} = 7$. It is a *perfect* code that can correct up to 3 errors, i.e., if 4 or more errors occur there will be a decoding error.

   (a) Find a good bound to the bit error probability $\mathrm{P_b}$ as a function of the bit SNR γ_{b} if we use the Golay code with BPSK modulation on an AWGN channel, with hard decision decoding at the receiver.

   (b) The *Golay* code has the following weight distribution for nonzero weight codewords:

weight	# codewords
7	253
8	506
11	1288
12	1288
15	506
16	253
23	1

   Find a good bound to the bit error probability $\mathrm{P_b}$ as a function of the bit SNR γ_{b} if we use the Golay code with BPSK modulation on an AWGN channel, with soft decision decoding at the receiver.

   (c) Plot your bounds on $\mathrm{P_b}$ (on a log scale) for hard and soft decision decoding as a function of γ_{b} for γ_{b} ranging from 2 dB to 10 dB and compare with uncoded BPSK.

2. [**Soft Decision Decoding with Binary Coherent Orthogonal Modulation in AWGN**]
In class we found the structure of the MPE solution to the soft decision decoding problem with BPSK in AWGN. Repeat the analysis for soft decision decoding with binary coherent orthogonal modulation in AWGN. Simplify the solution for the MPE estimate of the transmitted codeword index as much as possible.

3. [**Convolutional Codes**]
Consider the rate-$\frac{1}{2}$ convolution code studied in class with

$$c_\ell^{(1)} = b_\ell + b_{\ell-2}, \quad c_\ell^{(2)} = b_\ell + b_{\ell-1} + b_{\ell-2}.$$

   (a) Decode the received sequence $R = 10\ 01\ 00\ 01\ 01\ 10\ 00\ 01\ 11$ using a Viterbi decoder.

   (b) Now suppose the transmitted sequence is 11 01 00 01 00 10 10 00 01 11 00 . . . and it is received without error but somehow the first bit is lost. The Viterbi decoder is bravely, but misguidedly, trying to decode the misframed received sequence 10 10 00 10 01 01 00 00 11 10 . . . What are the path metrics in this case? Can the Viterbi decoder use the path metrics to detect the misframing?

   (c) As mentioned in class, practical implementations of the Viterbi decoder can only decode finite chunks of the received sequence at a time. At the end of the first chunk, the decoder picks the path with minimum distance, resets the starting state as the state which corresponds to mimimum distance path, and restarts the Viterbi algorithm. So for the decoding of the second chunk (and future chunks), the starting state may not be 0 state. Furthermore, the starting state assumed by the decoder may be different from the actual state of the encoder at that time.

Let us consider one such situation here. The decoder decides that the initial state is 10, while the encoder is actually in state 00. All paths considered by the decoder start at 10. Now suppose the encoder is encoding the *information* sequence 101000000... from this point on. Sketch the path through the trellis that is found by the decoder. You should see that the one of the decoder sequences and the encoded sequence merge pretty soon. What is the decoded information sequence? (You'll be surprised!)

4. **[Raised Cosine Pulse]**
The raised cosine spectrum is given by

$$X_{\mathrm{rc}}(f) = \begin{cases} T & 0 \le |f| \le \frac{1-\beta}{2T} \\ \frac{T}{2}\left[1 + \cos\left(\frac{\pi T}{\beta}\left(|f| - \frac{1-\beta}{2T}\right)\right)\right] & \frac{1-\beta}{2T} \le |f| \le \frac{1+\beta}{2T} \\ 0 & |f| > \frac{1+\beta}{2T} \end{cases}$$

Show that this pulse satisfies the Nyquist criterion for $0 \le \beta \le 1$.

5. **[MLSE]**
Consider the problem of BPSK communication on an ISI channel with:

$$\mathcal{E}_{\mathrm{s}} = 2 \quad \text{and} \quad h_0 = \sqrt{0.5},\ h_1 = 0.5\ h_2 = 0.5$$

Suppose the received sequence $(z_1, z_2, \ldots, z_6)$ is given by (2,0.5,-1,0,-2,3)

Find the MLSE estimate of the transmitted symbols. (This is similar to the example with did in class, except that we have a trellis with 4 states.