University of Illinois **Fall 2016**

ECE 562: Problem Set 4
Noncoherent Demodulation, Differential PSK, Error Control Coding

Due: Thursday, Oct 20 in class
Reading: Lecture Notes 10-13; Sections 4.5-4.6 and 7.1-7.6 Proakis & Salehi

1. **[Noncoherent Demodulation of Linearly Modulated Signals]**
The received signal for one symbol period for linear memoryless modulation on an ideal AWGN channel is given by:
$$r(t) = \sqrt{\mathcal{E}_m} e^{j\theta_m} g(t) e^{j\phi} + w(t)$$
where the phase offset ϕ is due to the delay introduced by channel. If ϕ is known at the receiver, we can correct for it (by projecting $y(t)$ on $g(t)e^{j\phi}$ to produce the sufficient statistic) and suffer no loss in detection performance. However, if ϕ is not known, we may project $r(t)$ on $g(t)$ to get the sufficient statistic
$$R = \sqrt{\mathcal{E}_m} e^{j\theta_m} e^{j\phi} + W$$
where $W \sim \mathcal{CN}(0, N_0)$. Since ϕ is not of direct interest to the receiver, we treat it as a nuisance parameter. As we saw in class, there are two ways to deal with such parameters.
   (a) Assume that $\phi \in [0, 2\pi]$, and find $\hat{m}_{\text{JML}}(r)$ using the joint ML approach. Interpret your answer.
   (b) Now assume that ϕ is a random variable that is uniformly distributed on $[0, 2\pi]$, and find $\hat{m}_{\text{MAP}}(y)$. Simplify your answer as much as possible (note that your answer can be written in terms of the Bessel function I_0).
   Note: You should see from this problem that noncoherent demodulation of linearly modulated signals is not a very good idea.

2. **[BPSK versus QPSK with Phase Error]**
We saw in the previous problem that noncoherent detection of linearly modulated signals is not a good idea. In this problem we consider the situation where the phase is estimated at the receiver (say, using a phase-locked loop) but there is a residual phase error ϕ, which is not known at the receiver. As before, the sufficient statistic for decision making is given by:
$$R = \sqrt{\mathcal{E}_m} e^{j\theta_m} e^{j\phi} + W$$
The receiver does not know that there is a phase error, and so it makes decisions assuming that ϕ is equal to zero. Clearly the presence of the phase error should affect the error probability at the output of the detector.
   (a) Show that for BPSK
$$\text{P}_\text{b} = Q\left(\sqrt{2\gamma_\text{b}\cos^2\phi}\right)$$
   (b) Show that for QPSK with Gray coding
$$\text{P}_\text{b} = \frac{1}{2}Q\left(\sqrt{4\gamma_\text{b}\sin^2(\phi+\pi/4)}\right) + \frac{1}{2}Q\left(\sqrt{4\gamma_\text{b}\cos^2(\phi+\pi/4)}\right)$$
   (c) Using Matlab or similar program, compare P_b for BPSK and QPSK for $\gamma_\text{b} = 10$ and $\phi = 0.1, 0.2, 0.3$ radians. Which modulation scheme is more sensitive to phase errors?

3. **[Closed-Form Expression for Error Probability for Noncoherent Orthogonal Signaling]**
In class we showed that the probability of correctly demodulating the transmitted symbol is given by:
$$\text{P}_\text{c} = \int_0^\infty x\left[1 - \exp\left(-\frac{x^2}{2}\right)\right]^{M-1} \exp\left[-\left(\frac{x^2}{2} + \gamma_\text{s}\right)\right] I_0\left(x\sqrt{2\gamma_\text{s}}\right)\, dx$$

(a) Use the binomial expansion on the second term in the integrand, and the fact that a Ricean pdf integrates to 1, to show that the symbol error probability is given by

$$\mathrm{P_e} = \sum_{n=1}^{M-1} (-1)^{n+1} \binom{M-1}{n} \frac{1}{n+1} \exp\left[-\frac{n\gamma_\mathrm{s}}{(n+1)}\right].$$

(b) Use part (a) to find an expression for $\mathrm{P_b}$ in terms of γ_b.

4. [**Asymptotic Performance of Noncoherent Orthogonal Signaling**]
In this problem you will show that noncoherent detection of orthogonally modulated signals suffer no performance loss relative to coherent detection as $M \to \infty$.

(a) Using the union bound, show that the following bound on $\mathrm{P_e}$ holds for M-ary orthogonal modulation with noncoherent detection:

$$\mathrm{P_e} < \frac{M}{2} \exp\left(-\frac{\gamma_\mathrm{b} \log_2 M}{2}\right)$$

(b) Now show that $\mathrm{P_e} \to 0$ and $M \to \infty$ as long as $\gamma_\mathrm{b} > 2\ln 2$. (Note that this asymptotic result is identical to the result we showed in class for coherent detection.)

5. [**Differential Detector and Noncoherent Orthogonal Modulation**]
Show that for binary DPSK, the non coherent detector based on the two-symbol interval statistic $\underline{Y}_n$ is the same as the differential detector (see page 28 of the notes).

6. [**Comparison of Modulation Schemes**]
In class we saw a figure from the book by Proakis that depicts a comparison of the spectral efficiency ρ versus bit SNR γ_b for various modulation schemes at a symbol error probability of $\mathrm{P_e} = 10^{-5}$. Your task in this problem is to produce a similar figure for a *bit* error probability of $\mathrm{P_b} = 10^{-5}$.

(a) First write a program to solve for maximum (Shannon) ρ as a function of γ_b and plot the curve on a log scale for ρ and with γ_b in dB. (Use the range of -1.6 dB to 20 dB for the γ_b axis.)

(b) Add to the figure points corresponding to M-ary PSK, $M = 2, 4, 8, 16$ (assume Gray coding and use NNA).

(c) Add to the figure points corresponding to PAM with $M = 2, 4, 8$ (assume Gray coding).

(d) Add points for coherent M-ary orthogonal signaling, $M = 2, 4, 8, 16, 32$ (use UB).

(e) Add points for noncoherent M-ary orthogonal signaling, $M = 2, 4, 8, 16$ (use UB).

7. [**Properties of Linear Binary Block Codes**]
Consider a (n, k) linear binary block code.

(a) Show that $\omega^\mathrm{H}_\mathrm{min} = d^\mathrm{H}_\mathrm{min}$.

(b) Show that the set of distances of a particular codeword to all other codewords is the same for each codeword, i.e., if we define

$$\mathcal{D}_\ell = \{d^\mathrm{H}(\underline{c}_\ell, \underline{c}_m) : m \neq \ell\}$$

then $\mathcal{D}_\ell = \mathcal{D}_1$ for all ℓ.

(c) From part (b), conclude that linear binary block codes have the symmetry property that the probability of codeword error, conditioned on $\underline{c}_i$ being transmitted, is the same for all i, assuming BPSK modulation in AWGN.