1. **[Simplex Signal Set]**
   Consider a set of $M$ orthogonal signal waveforms $\{s_m(t)\}_{m=0}^{M-1}$ that have energy $\mathcal{E}$. Define a new set of $M$ waveforms as
   \[
s'_m(t) = s_m(t) - \frac{1}{M} \sum_{\ell=0}^{M-1} s_\ell(t), \quad m = 0, 1, \ldots, M - 1.
   \]
   Show that the $M$ signal waveforms have equal energy, given by $(1 - \frac{1}{M}) \mathcal{E}$, and are equally correlated, with correlation coefficients $\rho_{km} = -\frac{1}{M-1}$.

2. **[Signal Constellation Optimization]**
   Consider the signal constellation shown below
   \[\text{\includegraphics{signal_constellation.png}}\]
   (a) Show that the measure of goodness $\zeta$ is given by
   \[
   \zeta = \begin{cases} 
   \frac{12}{1+a^2} & \text{if } a > a^* \\
   \frac{6(a^2-a\sqrt{2}+1)}{1+a^2} & \text{if } a \leq a^*
   \end{cases}
   \]
   with $a^* = (\sqrt{2} + \sqrt{6})/2$
   (b) Maximize $\zeta(a)$ over $a \geq 1$ to find the best constellation.
   (c) Compare the result in (b) with $\zeta$ for 8-ary PAM.

3. **[Competing Signal Constellations]**
   Compare 8-PSK and rectangular 8-QAM in terms of their measures of goodness. (There are two optimal choices of rectangular 8-QAM constellations, and you may choose either one of them in your comparison.)

4. **[Complex Random Vector]**
   For a complex random vector $\mathbf{Y}$, show the following properties of the covariance matrix:
   \[
   \Sigma = (\Sigma_I + \Sigma_Q) + j(\Sigma_{IQ} - \Sigma_{IQ})
   \]
   and pseudo-covariance matrix:
   \[
   \tilde{\Sigma} = (\Sigma_I - \Sigma_Q) + j(\Sigma_{IQ} + \Sigma_{IQ})
   \]
5. **[WGN in Complex Baseband]**
Consider the signal \( s(t) = [\sin(\pi t) + j \cos(\pi t)] \mathbb{1}_{0 \leq t \leq 1} \). Suppose this signal is corrupted by complex WGN \( w(t) \) with PSD \( N_0 = 2 \) to form the received signal
\[
r(t) = s(t) + w(t).
\]
Further suppose we form the random variable \( Z \) as:
\[
Z = \int_{0}^{1} r(t) \sin(\pi t) \, dt
\]
(a) Find \( P\{Z_I \geq 2\} \).
(b) Find \( P\{Z_I + 2Z_Q \geq 1\} \).
(c) Find \( P\{Z_I \geq 1, Z_Q \geq 2\} \).

6. **[Unequal Priors]**
Describe how the optimal decision regions for BPSK signaling get modified when the priors on the messages are not the same, say \( \pi_0 = 3/4 \) and \( \pi_1 = 1/4 \).

7. **[Probability of error for PAM]**
Compute (the exact) \( P_e \) as a function of \( E_s \) for 8-ary PAM. Note that \( P_{e,m} \) is not the same for all \( m \) in this case.