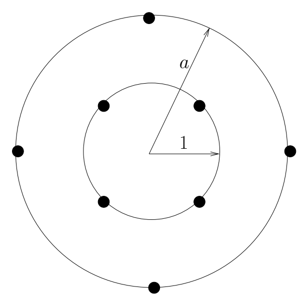
University of Illinois **Fall 2016**

ECE 562: Problem Set 2
Digital Modulation, Complex WGN, Optimum Receiver in WGN

Due: Tuesday September 20 in class
Reading: Lecture Notes 5-8; Sections 3.1,3.2,4.1 of Proakis & Salehi; Sections 3.3, 2.5, 2.6. of Madhow, Chapter 4 of Wozencraft & Jacobs.

1. [**Simplex Signal Set**]
Consider a set of M orthogonal signal waveforms $\{s_m(t)\}_{m=0}^{M-1}$ that have energy $\mathcal{E}$. Define a new set of M waveforms as
$$s'_m(t) = s_m(t) - \frac{1}{M}\sum_{\ell=0}^{M-1} s_\ell(t)\,, \quad m = 0, 1, \ldots, M-1\,.$$
Show that the M signal waveforms have equal energy, given by $\left(1 - \frac{1}{M}\right)\mathcal{E}$, and are equally correlated, with correlation coefficients $\rho_{km} = -\frac{1}{M-1}$

2. [**Signal Constellation Optimization**]
Consider the signal constellation shown below



   (a) Show that the measure of goodness ζ is given by
$$\zeta = \begin{cases} \frac{12}{1+a^2} & \text{if } a > a^* \\ \frac{6(a^2 - a\sqrt{2}+1)}{1+a^2} & \text{if } a \le a^* \end{cases}$$
   with $a^* = (\sqrt{2} + \sqrt{6})/2$

   (b) Maximize $\zeta(a)$ over $a \ge 1$ to find the best constellation.

   (c) Compare the result in (b) with ζ for 8-ary PAM.

3. [**Competing Signal Constellations**]
Compare 8-PSK and rectangular 8-QAM in terms of their measures of goodness. (There are two optimal choices of rectangular 8-QAM constellations, and you may choose either one of them in your comparison.)

4. [**Complex Random Vector**]
For a complex random vector $\underline{Y}$, show the following properties of the covariance matrix:
$$\Sigma = (\Sigma_I + \Sigma_Q) + j(\Sigma_{QI} - \Sigma_{IQ})$$
and pseudo-covariance matrix:
$$\check{\Sigma} = (\Sigma_I - \Sigma_Q) + j(\Sigma_{QI} + \Sigma_{IQ})$$

5. **[WGN in Complex Baseband]**
Consider the signal $s(t) = [\sin(\pi t) + j\cos(\pi t)]\mathbb{1}_{0 \le t \le 1}$. Suppose this signal is corrupted by complex WGN $w(t)$ with PSD $N_0 = 2$ to form the received signal

$$r(t) = s(t) + w(t) .$$

Further suppose we form the random variable Z as:

$$Z = \int_0^1 r(t)\sin(\pi t)dt$$

   (a) Find $\mathsf{P}\{Z_I \ge 2\}$.
   (b) Find $\mathsf{P}\{Z_I + 2Z_Q \ge 1\}$.
   (c) Find $\mathsf{P}\{Z_I \ge 1, Z_Q \ge 2\}$.

6. **[Unequal Priors]**
Describe how the optimal decision regions for BPSK signaling get modified when the priors on the messages are not the same, say $\pi_0 = 3/4$ and $\pi_1 = 1/4$.

7. **[Probability of error for PAM]**
Compute (the exact) $\mathrm{P_e}$ as a function of $\mathcal{E}_\mathrm{s}$ for **8-ary** PAM. Note that $\mathrm{P}_{\mathrm{e},m}$ is not the same for all m in this case.