1. **Chernoff and Bhattacharya Bounds**
   Consider the binary hypothesis testing problem with
   \[ p_0(y) = \frac{1}{2}e^{-|y|} \quad \text{and} \quad p_1(y) = e^{-2|y|} \]
   Assume equal priors.
   (a) Find \( P_e \) for Bayes rule.
   (b) Find the Bhattacharya bound on \( P_e \).
   (c) Find the Chernoff bound on \( P_e \).

2. **Chernoff Information and K-L Divergence**
   Recall that the Chernoff information for \( p_0 \) and \( p_1 \) is given by
   \[ C(p_0, p_1) \triangleq \max_{u \in [0,1]} - \ln \int_Y p_0^{1-u}(y)p_1^u(y)d\mu(y) \]
   Now define the “geometric mixture” of \( p_0 \) and \( p_1 \) by:
   \[ p_u(y) \triangleq \frac{p_0^{1-u}(y)p_1^u(y)}{\int_Y p_0^{1-u}(y)p_1^u(y)d\mu(y)} \]
   Show that the optimizing value of \( u \) in definition of \( C(p_0, p_1) \) satisfies the equation:
   \[ C(p_0, p_1) = D(p_u \| p_0) = D(p_u \| p_1) \]
   Hint: You may want to use the fact that \( p_u(y) \) can be written as:
   \[ p_u(y) = \frac{p_0(y) L(y)^u}{E[L(Y)^u]} \]

3. **Slight Generalization of Cramer’s Theorem**
   Let \( X_1, X_2, \ldots, \) be i.i.d. with mean \( \mathbb{E}[X] \), and let \( S_n = \sum_{k=1}^{n} X_k \). Then, for \( a > \mathbb{E}[X] \), we know from class that
   \[ \Pr\{S_n \geq na\} \leq \exp(-n\Lambda_X(a)) \]
   and that, given \( \epsilon > 0 \), there exists \( n_\epsilon \) such that
   \[ \Pr\{S_n \geq na\} \geq \exp(-n(\Lambda_X(a) + \epsilon)) \quad \text{for all} \quad n > n_\epsilon. \]
   These two bounds establish Cramér’s Theorem. Now use these bounds and the continuity of \( \Lambda_X \) to show the following generalization of this result. Suppose \( a_n \to a \) as \( n \to \infty \) and \( a > \mathbb{E}[X] \), then
   \[ \lim_{n \to \infty} \frac{1}{n} \ln \Pr\{S_n \geq na_n\} = -\Lambda_X(a) \]

4. **Gaussian – I**
   Consider the detection problem where the observations, \( \{Y_k, k \geq 1\} \) are i.i.d. \( N(\mu_j, \sigma^2) \) random variables under \( H_j, j = 0, 1. \)
(a) Evaluate the cumulant generating function \( \kappa_0(u) \).
(b) Use \( \kappa_0(u) \) to find \( D(p_0 \| p_1) \) and \( D(p_1 \| p_0) \).
(c) Find the rate functions \( \Lambda_0(\tau) \) and \( \Lambda_1(\tau) \).
(d) Find the Chernoff information \( C(p_0, p_1) \).
(e) Solve the Hoeffding problem when the constraint on the false alarm exponent is equal to \( \gamma \), for \( \gamma \in (0, D(p_1 \| p_0)) \).

5. [Gaussian - II]
Repeat parts (a)-(d) of Problem 4 for the case where the observations, \( \{Y_k, k \geq 1\} \) are i.i.d. \( \mathcal{N}(0, \sigma_j^2) \) random variables under \( H_j, j = 0, 1 \).