University of Illinois at Urbana-Champaign

# ECE 559BH: Topics in Communications: Distributed Network Algorithms

**Spring 2006**
**Midterm Exam**

**Thursday, March 30, 1-2:45 p.m.**

Name: _____

- You have 105 minutes for this exam. The exam is open notes and open book.
- Write your answers in the spaces provided.
- **Please show all of your work. Answers without appropriate justification will receive very little credit.** If you need extra space, use the back of the previous page.

Score:

1. _____ (10 pts.)

2. _____ (10 pts.)

3. _____ (10 pts.)

Total: _____(30 pts.)

**Problem 1** Consider the online paging problem with cache size $k$ and $n$ total memory items, with $1 \leq k < n$, and suppose the requests are random and independent, with each request being uniformly distributed over the $n$ memory items.

(a) What is the miss rate (misses per request in the long run) for any online paging algorithm?

(b) Recall that the MARKER algorithm works in rounds. What is the expected number of misses of the MARKER algorithm per round? (Solve independently of (a)).

(c) What is the expected number of requests per round of the MARKER algorithm? (Solve independently of (a)).

(d) How are the answers to (a), (b), and (c) related? (This does not require knowledge of the MARKER algorithm.)

**Problem 2** Recall that in each step of Luby's LT decoding process an output symbol with reduced degree one (i.e. a symbol from the gross ripple) is selected and processed. It is decoded, its corresponding input symbol is subtracted from the other output symbols containing it, and the degrees of those output symbols are reduced by one. The decoding process ends when the gross ripple is empty at the end of some step. Let $A_o$ be the set of input symbols that are not decoded after the process ends. Show that $A_o$ is completely determined by the set of output symbols initially given to the decoder. That is, $A_o$ does not depend on which output symbol is chosen from the ripple at each step. (Hint: It may be helpful to characterize $A_o$ in a way that is independent of the choices made by the decoding algorithm.)

**Problem 3** Given $1 \leq d < n$, let $U = \{u_1, \ldots, u_n\}$ and $V = \{v_1, \ldots, v_n\}$ be disjoint sets of cardinality $n$, and let $G$ be a bipartite random graph with vertex set $U \cup V$, such that if $V_i$ denotes the set of neighbors of $u_i$, then $V_1, \ldots, V_n$ are independent, and each is uniformly distributed over the set of all $\binom{n}{d}$ subsets of $V$ of cardinality $d$. A matching for $G$ is a subset of edges $M$ such that no two edges in $M$ have a common vertex. Let $Z$ denote the maximum of the cardinalities of the matchings for $G$.
(a) Find bounds $a$ and $b$, with $0 < a \leq b < n$, so that $a \leq E[Z] \leq b$.

(b) Give an upper bound on $P\{|Z - E[Z]| \geq \gamma\sqrt{n}\}$, for $\gamma > 0$, showing that for fixed $d$, the distribution of $Z$ is concentrated about its mean as $n \to \infty$.

(c) Suggest a greedy algorithm (similar to the LT process) for finding a large cardinality matching.