University of Illinois at Urbana-Champaign

ECE 559BH: Topics in Communications: Distributed Network Algorithms

Spring 2006
Final Exam

Friday, May 5, 1:30-3:30 p.m.

Name: ______________________________

- You have 120 minutes for this exam. The exam is open notes and open book.
- Write your answers in the spaces provided.
- **Please show all of your work. Answers without appropriate justification will receive very little credit.** If you need extra space, use the back of the previous page.

Score:

1. __________ (30 pts.)

2. __________ (30 pts.)

Total: ________(60 pts.)

Problem 1 Consider a relay network with node 1 being the source node, node 2 the relay node, and node three the terminal node. Suppose for $i \in \{1, 2\}$, that node i sends a coded symbol in each time slot, and each transmission is received without error at the next node with some fixed probability z_i, and is erased with probability $1 - z_i$. The symbols sent by node 1 are generated by LT coding, based on K input symbols in $\{0, 1\}^L$ for $L >> 1$, and a robust soliton degree distribution. Node 2 in each slot sends the coded symbol that it most recently received from node 1. Node 3 is able to signal back to the other two nodes when it has received $K(1 + \epsilon)$ symbols, allowing it to decode successfully, but no other feedback is available.

(a) What is the probability that node 3 receives at least one copy of a given output symbol? (Hint: This is the throughput, measured in symbols per slot, if ϵ and packet headers are ignored. If $z_1 = z_2 = 1/2$, the throughput is $1/3$.)

(b) Suppose the strategy is changed at node 2, so that node 2 continues to send one of the packets it received in each slot, but the symbols is chooses to send can be chosen differently. Give a choice rule leading to a higher information rate than the one given. In particular, show that throughput $3/8$ is possible if $z_1 = z_2 = 1/2$ (again, ignoring factors arbitrarily close to one).

(c) What information rate is achievable if node 2 is allowed to combine the symbols it receives to make new ones? Explain.

Problem 2 (Variation of Mitzenmacher and Upfal, problem 5.11) A system initially with n balls and n bins operates in rounds. In each round, each remaining ball is thrown into a bin. Any ball landing in a bin which no other ball falls into during the round, is removed. The other balls are carried over to the next round.

(a) Let $g(b)$ denote the number of balls remaining at the end of a round, given there are b balls at the start of a round. Find $g(b)$ and show that $g(b) \leq b^2/n$.

(b) Let $x_0 = n$ and $x_{j+1} = g(x_j)$ for $j \geq 1$. Thus, x_j refers to the number of balls left after j rounds, under the approximation that the outcome of each round exactly follows expectations. Let $\tau = \min\{j : x_j \leq 1\}$. Show that $\tau = O(\ln \ln n)$ as $n \to \infty$.

(c) Suppose the number of balls at the beginning of a round is a random variable B, which is Poisson distributed with mean b. Describe the probability distribution of the number of balls left at the end of the round.

(d) (10 points extra credit) Prove that if c is sufficiently large and σ is the (random) number of rounds required until all balls are removed, then $P\{\sigma \geq c \ln \ln n\} \to 0$ as $n \to \infty$.