EXAM 3 SOLUTIONS
Monday, December 15, 2014

Problem 1  (25 points)

(a) \( F_n(\epsilon) \) is the probability that either there is no datum in the range \([\theta - \epsilon, \theta]\), or there is no datum in the range \([\theta, \theta + \epsilon]\), or both. This probability is

\[
F_n(\epsilon) = 2(1 - \epsilon)^n - (1 - 2\epsilon)^n
\]

(b) After \( n \) training data we are guaranteed that \( b_{n+1} \leq \theta \leq b_{n+1} + s_n \). Given this condition, \( F_n(\epsilon) \) is the probability that \( b_{n+1} + \epsilon \leq \theta \) or \( \theta \leq b_{n+1} + s_n - \epsilon \) or both, which is

\[
F_n(\epsilon) = \begin{cases} 
1 & \epsilon \leq 2^{-(n+1)} \\
1 - 2^{(n+1)}\epsilon & 2^{-(n+1)} \leq \epsilon \leq 2^{-n} \\
0 & 2^{-n} \leq \epsilon 
\end{cases}
\]

Problem 2  (25 points)

(a)

\[
\frac{\partial E}{\partial \mu_k} = 2 \sum_{i: \hat{k}_i = k} (\mu_k - x_i)
\]

Setting the derivative equal to zero yields the desired result.

(b) The index will change if \( \Delta F < 0 \), where

\[
\Delta F = \begin{cases} 
\frac{\lambda}{n_{k_i}+1} + \frac{\lambda}{n_{\hat{k}_i}} + \frac{\lambda}{n_{k_i}+1} + \frac{\lambda}{n_{\hat{k}_i}} & x_i \text{ unlabeled, or } y_i = y(k_i) = y(\hat{k}_i) \\
\frac{\lambda}{n_{k_i}} + \frac{\lambda}{n_{\hat{k}_i}} & y_i = y(\hat{k}_i) \neq y(k_i) \\
\frac{\lambda}{n_{k_i}} + \frac{\lambda}{n_{\hat{k}_i}} & y_i \neq y(k_i) \text{ and } y_i \neq y(\hat{k}_i)
\end{cases}
\]

where \( y(k_i) \) is the majority label of cluster \( k_i \) after reassignment, \( y(\hat{k}_i) \) is the majority label of cluster \( \hat{k}_i \) before reassignment, and \( n_{k_i} \) and \( n_{\hat{k}_i} \) are the counts of those clusters before reassignment.

Problem 3  (25 points)
(a) 

\[ \gamma_i(h; \theta) = \frac{c_h \lambda_h e^{-\lambda_h v_i}}{\sum_{k=1}^{m} c_k \lambda_k e^{-\lambda_k v_i}} \]

(b) Define 

\[ \lambda_h = \frac{\sum_{i=1}^{n} \gamma_i(h; \hat{\theta})}{\sum_{i=1}^{n} v_i \gamma_i(h; \hat{\theta})} \]

Problem 4  (25 points)

(a) 

\[ \epsilon_{i\ell} = -\frac{t_{i\ell}}{z_{i\ell}} g'(b_{i\ell}) \]

(b) 

\[ \frac{\partial \mathcal{E}}{\partial v_{\ell k}} = \sum_{i=1}^{n} \epsilon_{i\ell} y_{ik} \]

\[ \frac{\partial \mathcal{E}}{\partial u_{kj}} = \sum_{i=1}^{n} f'(a_{ik}) x_{ij} \sum_{\ell=1}^{r} \epsilon_{i\ell} v_{\ell k} \]