UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 544NA Pattern Recognition
Fall 2014

EXAM 3 SOLUTIONS

Monday, December 15, 2014

Problem 1 (25 points)

(a) $F_n(\epsilon)$ is the probability that either there is no datum in the range $[\theta-\epsilon,\theta]$, or there is no datum in the range $[\theta,\theta+\epsilon]$, or both. This probability is

$$F_n(\epsilon) = 2(1-\epsilon)^n - (1-2\epsilon)^n$$

(b) After n training data we are guaranteed that $b_{n+1} \le \theta \le b_{n+1} + s_n$. Given this condition, $F_n(\epsilon)$ is the probability that $b_{n+1}+\epsilon \le \theta$ or $\theta \le b_{n+1}+s_n-\epsilon$ or both, which is

$$F_n(\epsilon) = \begin{cases} 1 & \epsilon \le 2^{-(n+1)} \\ 1-2^{(n+1)}\epsilon & 2^{-(n+1)} \le \epsilon \le 2^{-n} \\ 0 & 2^{-n} \le \epsilon \end{cases}$$

Problem 2 (25 points)

(a)

$$\frac{\partial \mathcal{E}}{\partial \mu_k} = 2 \sum_{i:k_i=k} (\mu_k - x_i)$$

Setting the derivative equal to zero yields the desired result.

(b) The index will change if $\Delta\mathcal{F} < 0$, where

$$\Delta\mathcal{F} = \begin{cases} \|x_i-\mu_{k_i}\|^2 - \|x_i-\mu_{\hat{k}_i}\|^2 & x_i \text{ unlabeled, or } y_i = y(k_i) = y(\hat{k}_i) \\ \frac{\lambda}{n_{k_i}+1} + \|x_i-\mu_{k_i}\|^2 - \|x_i-\mu_{\hat{k}_i}\|^2 & y_i = y(\hat{k}_i) \ne y(k_i) \\ \|x_i-\mu_{k_i}\|^2 - \|x_i-\mu_{\hat{k}_i}\|^2 - \frac{\lambda}{n_{\hat{k}_i}} & y_i = y(k_i) \ne y(\hat{k}_i) \\ \frac{\lambda}{n_{k_i}+1} + \|x_i-\mu_{k_i}\|^2 - \|x_i-\mu_{\hat{k}_i}\|^2 - \frac{\lambda}{n_{\hat{k}_i}} & y_i \ne y(k_i) \text{ and } y_i \ne y(\hat{k}_i) \end{cases}$$

where $y(k_i)$ is the majority label of cluster k_i after reassignment, $y(\hat{k}_i)$ is the majority label of cluster $\hat{k}_i$ before reassignment, and n_{k_i} and $n_{\hat{k}_i}$ are the counts of those clusters before reassignment.

Problem 3 (25 points)

(a)

$$\gamma_i(h;\theta) = \frac{c_h \lambda_h e^{-\lambda_h v_i}}{\sum_{k=1}^{m} c_k \lambda_k e^{-\lambda_k v_i}}$$

(b) Define

$$\lambda_h = \frac{\sum_{i=1}^{n} \gamma_i(h;\hat{\theta})}{\sum_{i=1}^{n} v_i \gamma_i(h;\hat{\theta})}$$

Problem 4 (25 points)

(a)

$$\epsilon_{i\ell} = -\frac{t_{i\ell}}{z_{i\ell}} g'(b_{i\ell})$$

(b)

$$\frac{\partial \mathcal{E}}{\partial v_{\ell k}} = \sum_{i=1}^{n} \epsilon_{i\ell} y_{ik}$$

$$\frac{\partial \mathcal{E}}{\partial u_{kj}} = \sum_{i=1}^{n} f'(a_{ik})\, x_{ij} \sum_{\ell=1}^{r} \epsilon_{i\ell} v_{\ell k}$$