UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 544NA Pattern Recognition

Homework 7

Fall 2013

Assigned: Thursday, October 31, 2013 Due: Thursday, November 14, 2013

Reading: NNPR Chapter 8, links posted on the class website

Problem 7.1

A kernel $K(x, y)$ is called a Mercer kernel if it satisfies the Mercer conditions: 1) $K(x, y)$ is symmetric, 2) continuous, and 3) positive semi-definite. Prove the following results

(a) If K_1 and K_2 are Mercer kernels, then their product, $K = K_1 K_2$ is also a Mercer kernel.

(b) If K_1 and K_2 are Mercer kernels, then their linear combination, $K = aK_1 + bK_2$, $a \geq 0$ and $b \geq 0$ is also a Mercer kernel.

(c) $K(x, y) = (x^T y + c)^d$ is a Mercer kernel for any $c \geq 0$ and for $d = 1, 2, ...$ [Hint: use the results from (a) and (b)].

(d) Let $x, y \,\epsilon\, \{1, 2, ..., 100\}$. Show that $K(x, y) = min(x, y)$ is a positive-definite discrete kernel. That is, $\sum_{x,y} f(x)f(y)K(x, y) \geq 0$ for any nonzero sequence $f(x)$, $x = 1, 2, ..., 100$.

Matlab Exercises

Problem 7.2

Again, for the diabetes dataset, create a training set and a test set as you did in HWs 4, 5, 6. You are expected to turn in original code for each part, but you may reuse your own code from previous assignments.

(a) Use PCA to reduce the dimensionality of the training set to *two* features; train a linear classifier on the new, lower dimensional training set. Test your classifier on the test set (use the transformations learned on the *training set* to project the test set onto the lower dimensional space).

(b) Learn the weights of an autoencoder (implemented with a restricted Boltzmann machine) with two nodes in the hidden layer (excluding bias) on the training set; train a linear classifier on the lower-dimensional space represented by the hidden nodes. Test your classifier on the test set.

(c) Implement kernel PCA with a Gaussian kernel to reduce the dimensionality of the training set to *two* features. As before, learn a linear classifier on the training set and test the classifier on the test set.

(d) Comment on your results, comparing them to each other and also to a linear classifier trained on the original feature space.