Chapter 5
Concept learning (binary)
realizable \((X, C, P)\)

Given samples \((x_1, y_1), \ldots, (x_n, y_n) \ y_i \in \{0, 1\}\)

\[ L_p(C', C^*) = P(C' \Delta C^*) \]

Say \( C' \) is \( \varepsilon \)-accurate (for \( P, C^* \)) if

\[ P(C' \Delta C^*) \leq \varepsilon. \]

Given \((X, C, P)\)

Definition. A learning algorithm

\[ A : A_n((x_1, y_1), \ldots, (x_n, y_n)) \rightarrow \hat{C} \] is

Probably Almost Correct (PAC) if for any \( \varepsilon, \delta > 0 \), there exists \( n(\varepsilon, \delta) \) so

- for any \( C, C' \) for any \( C \), if \( n \geq n(\varepsilon, \delta) \) then \( \hat{C} \) is \( \varepsilon \)-accurate
- and for any \( P, C \)
with probability at least $1 - \delta$.

**Theorem 5.1** If $|\mathcal{C}| < \infty$ then there exists a PAC algorithm (i.e. the problem is PAC learnable).

**Proof**
Let $\mathcal{C} = \{C_1, \ldots, C_M\}$

Know $C^* \in \mathcal{C}$.

Fix $\mathcal{P}, \mathcal{E}$.

Fix $\delta, \epsilon > 0$.

Let

$$\beta = \{C \in \mathcal{C} : P(C \neq C^*) > \epsilon\}$$

- "bad" concepts

$$P(C = C_3) \leq (1 - \epsilon)^n$$

$\hat{C} = A(x_1, y_1, \ldots, x_n, y_n)$

So if $\mathcal{C} \subseteq (1 - \epsilon)^n < \delta$ then

$P(\hat{C} \text{ is } \epsilon\text{-accurate}) \geq 1 - \delta$. 

Example - Axis parallel rectangles, Realizable case

\[ X = [0,1]^2 \quad \mathcal{C} = \{ \text{axis parallel rectangles} \} \]

\( P = \text{all prob. dists on } X \)

Let

\( \hat{C} \) be the smallest rectangle giving zero errors on test data.

Claim This is a PAC algorithm.
New picture. Drew $C^*$ = true rectangle

Done if $P(V, U, V_2, U, H_1, U, H_2) \leq \varepsilon$

with probability at least $1 - \delta$.

Newer picture
$P, C'$ fixed.

Select $a$ so $[a, q] \times [a, b_a]$ is smallest rectangle of this form with $P$ probability $\geq \frac{3}{4}$

$$P([a, q] \times [a, b_a]) \geq \frac{3}{4} \equiv P([a, a] \times [a, b_a])$$

If $4 \left(1 - \frac{3}{4}\right)^n \leq 8$

then $P(U_1 U_2 U_1 U_2 U_1 U_2 U_1 U_2) \leq \varepsilon$ with prob. at least $1 - \delta$. \hfill \square$
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X = [0, 1]

G : \xi \in [0, 1] : 0 \leq a \leq 1

\emptyset - all dist's on [0, 1]