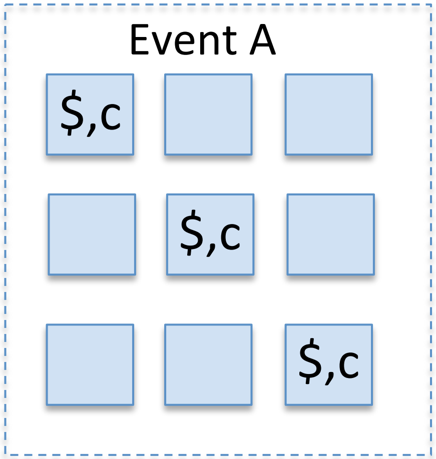
A Probabilistic Approach to the Monty Hall Problem

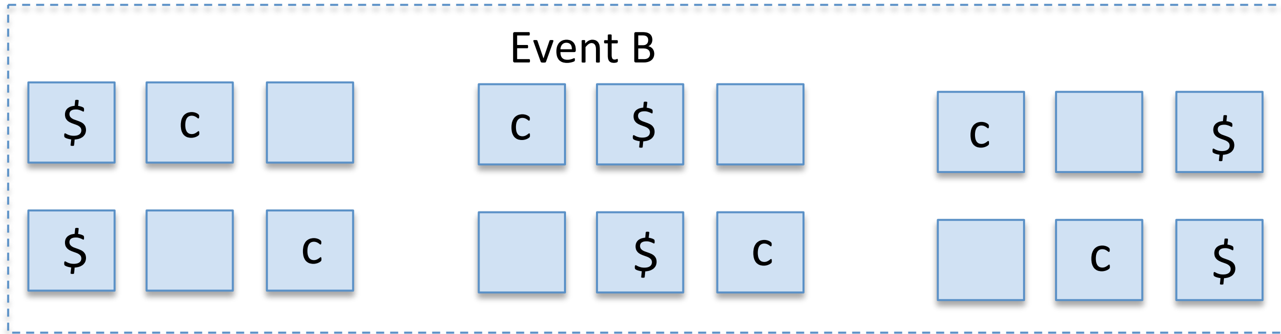
The Problem

- Three boxes
- Unseen by you, one is chosen uniformly, and has $20 placed inside
- You select a box, but do not open it.
- Monty Hall opens a box, shows you it is empty.
- You are given the opportunity to change your selection….what is the probability of winning if you do?

The Approach

Experiment is sampling $ box, choosing box





3 outcomes

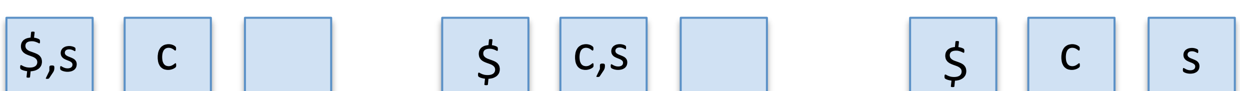
- Each has probability (1/3)*(1/3) = 1/9

6 outcomes

- Each has probability (1/3)*(1/3) = 1/9

Probability of winning if choice changed

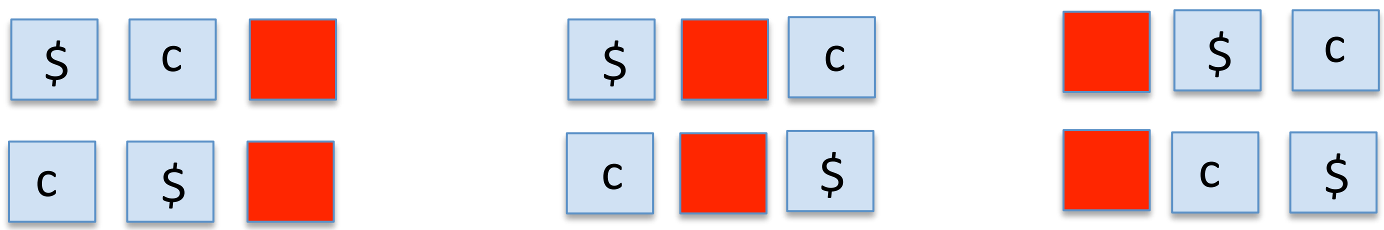
- 6 events where changing choice *might* win
  - Refine each event into 3 which reflect 2^{nd} choice (s)
  - For each, P(win|changed selection) = 1/2
  - Sum of probabilities of events reflecting win given change : 6*(1/9)*(1/2) = 1/3



The Approach

Now Monty Hall reveals an empty box

Outcomes in Event B



Observe---when MH shows a box, the money has to be in the "other box"

Event C = outcomes where box left unchosen and unrevealed contains $

$$
\begin{aligned}
P(C) &= P(B) * P(C|B) + P(B^c) * P(C|B^c) \\
&= P(B) * P(C|B) \\
&= P(B) \\
&= 2/3
\end{aligned}
$$