CTMC Example (client & web server) + Solution HW #2
9/30/16
State
1. "think" before transmit
2. transmit on network
3. serve request
4. client retry
5. transmit response

\( \lambda_t \): transmit request rate
\( \lambda_r \): transmission retry rate
\( \lambda_N, \lambda_F \) solutions to
\( \lambda_N + \lambda_F = \lambda_r \), \( \rho_f = \frac{\lambda_F}{\lambda_N + \lambda_F} \)
\( \lambda_S, \lambda_{SF} \) solutions to
\( p_{sf} = \frac{\lambda_{SF}}{\lambda_s + \lambda_{SF}} \), \( \lambda_s + \lambda_{SF} = \lambda_w \)

\( p_f = P(\text{network transmission fault}) \)
\( \frac{1}{\lambda_p} = \text{mean network transmission delay} \)

\( p_{sf} = P(\text{web service failure}) \)
\( \frac{1}{\lambda_w} = \text{mean web service response time} \)
Choose transition parameters to achieve desired holding time

- competing processes view

\[ P(\text{success}) = \frac{\lambda_s}{\lambda_s + \lambda_f} \]

holding time is exponential, rate \( \lambda_s + \lambda_f \)

IF we want the holding time to reflect time passage with success or failure occurring after this passage then we need to set \( \lambda_s, \lambda_f \) so that

\[ \lambda_s + \lambda_f = \lambda_p, \text{ desired rate} \]

so if failure prob is \( p_f \) and \( \lambda_p \) is given

\[ p_f = \frac{\lambda_f}{\lambda_s + \lambda_f} \quad \text{and} \quad \lambda_s + \lambda_f = \lambda_p \] solved gives \( \lambda_s, \lambda_f \)
Model of Client-Server with network and request failures

State 1: Client requests transmission with rate $\lambda_t$

State 2: Client transmits into network. $\lambda_N$ and $\lambda_F$ chosen so that chance of failure is $\lambda_F/(\lambda_N + \lambda_F)$, and mean transmission time is

$$\frac{1}{\lambda_p} = \frac{1}{\lambda_F + \lambda_N}$$

State 3: Server handles request. $\lambda_s$ and $\lambda_{sf}$ chosen so that chance of failure is $\lambda_{sf}/(\lambda_s + \lambda_{sf})$ and mean service time is

$$\frac{1}{\lambda_{st}} = \frac{1}{\lambda_s + \lambda_{sf}}$$

State 4: Host detected failed request, retries with rate $\lambda$
QUESTIONS

* what is Q matrix?

* what is successful request rate?

$$\Pi_1 \cdot \lambda_c$$

* what is network utilization? (Fraction of time network active)

$$\Pi_2 + \Pi_5$$

* what is probability of request succeeding w/o error?

$$\left( \frac{\lambda_N}{\lambda_N + \lambda_F} \right) \left( \frac{\lambda_S}{\lambda_S + \lambda_{SF}} \right) \left( \frac{\lambda_N}{\lambda_N + \lambda_F} \right)$$

* Distribution of number of transmission attempts to success?

* mean time to complete request?

Not so easy from here
Homework #2 Solution

\[ P_{k,k+1} \]

happens when exactly 1 backed off host sends, and no offering load hosts send

\[
= (1 - p_s)^k \cdot (N - k) \cdot p_r \cdot (1 - p_r)^{N-k-1}
\]

0 offering load \hspace{1cm} 1 backed off

\[ P_{k,k-1} \]

happens when 1 offering load host sends and at least 1 backed off host sends

\[
= k \cdot p_s \cdot (1 - p_s)^{k-1} \cdot (1 - (1 - p_r)^k)^{N-k} \cdot \text{no backed off hosts}
\]

1 offering load \hspace{1cm} \text{At 1 backed off host}
\[ j > 1 \quad P_{k, k-j} \quad \text{exactly } j \text{ offering load hosts send,} \]  
\[ \text{what backed off hosts do does not matter} \]  
\[ = \binom{k}{j} p_s^j (1-p_s)^{k-j} \]

\[ P_{k, k} \quad \text{Cases} \quad (a) \text{ No offered load host sends, and no backed off host sends successfully} \]  
\[ \Rightarrow 0, \text{ or } >1 \text{ try} \]

\[ (b) \text{ Exactly } 1 \text{ offered load host sends and exactly } 0 \text{ backed off hosts send} \]

\[ (1-p_s)^k \cdot (1-(N-k)\cdot p_r \cdot (1-p_r)^{N-k-1}) + k \cdot p_s \cdot (1-p_s)^{k-1} \cdot (1-p_r)^{N-k} \]

0 offering 1 backed off host 1 offering 0 backed off load 0, >1 backed off hosts
Throughput

Count transmission when

(a) Retransmission succeeds, or

(b) $k \rightarrow k$ transition and offered load succeeds

\[
\sum_{k=0}^{N-1} \prod_k \cdot P_{k,k+1} + \sum_{k=0}^{N} \prod_k \cdot P(1 \text{ off. load host sends state remains } k)
\]

\[
= k \cdot p_s \cdot (1 - p_s) \cdot (1 - p_r)^{N-k} + (1 - p_s) \cdot (1 - (N-k) \cdot p_r \cdot (1 - p_r)^{N-k-1})
\]