ECE 536
Integrated Optics and Optoelectronics

TuTh 11:00-12:20, 3020 ECEB
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Office Hours: Tuesday 1:00-2:00 pm
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**Subject to Change**
Today’s Discussion

- Quantum Well Lasers and Scaling Laws
- Assignments
- Topics for Next Lecture
Quantum Well Lasers and Scaling Laws
Zero-Temperature Gain Spectrum

At $T = 0K$: $f_c(E) = \begin{cases} \frac{1}{2} & E < F_c \\ 0 & E > F_c \end{cases}$

Electron Concentration:

$$n = \frac{m^*_e}{\pi \hbar^2 L_z} \sum_{n \text{ occupied subbands}} (F_c - E_{en})$$

For Single Occupied State:

$$n = \frac{m^*_e}{\pi \hbar^2 L_z} (F_c - E_{e1}) \quad \text{and} \quad p = \frac{m^*_h}{\pi \hbar^2 L_z} (E_{h1} - F_v)$$

For the case where $n \sim p$, if $m^*_h > m^*_e$ then:

$$(F_c - E_{e1}) > (E_{h1} - F_v)$$

Gain:

$$g(\hbar \omega) = \begin{cases} g_{\text{max}} & E_{h1} < \hbar \omega < E_g + F_c - F_v \\ -g_{\text{max}} \sum_{n=m} H(\hbar \omega - E_{hm}) & \text{otherwise} \end{cases}$$

i.e. sum over all bands for which $F_c > E_{en}$

Quasi-Fermi level is further into the conduction band.
Finite Temperature Gain Spectrum

Define \( n_c = \frac{m_e^* k_B T}{\pi \hbar^2 L_z} \) and \( n_v = \frac{m_h^* k_B T}{\pi \hbar^2 L_z} \)

Electron Concentration:
\[
n = \sum_{n=1}^{\infty} \int_0^{\infty} \rho_e^{2D} (E) f_c^n (E) dE = \sum_{n=1}^{\infty} n_c \ln \left[ 1 + e^{(F_c - E_{en})/k_B T} \right]
\]

Hole Concentration:
\[
p = \sum_{m=1}^{\infty} n_v \ln \left[ 1 + e^{(E_{hm} - F_v)/k_B T} \right]
\]

Sum over both hh and lh bands

Gain Spectrum (single subband):
\[
g(h\omega) = g_{\max} \left[ f_c (\hbar\omega - E_{h1}^{e1}(0)) - f_v (\hbar\omega - E_{h1}^{c1}(0)) \right]
\]

Peak Gain:
\[
g_p = g_{\max} \left[ f_c (\hbar\omega = E_{h1}^{e1}) - f_v (\hbar\omega = E_{h1}^{c1}) \right]
\]

where \( f_c (\hbar\omega = E_{h1}^{e1}) = \frac{1}{1 + e^{(E_{e1} - F_c)/k_B T}} \)

and \( f_v (\hbar\omega = E_{h1}^{c1}) = \frac{1}{1 + e^{(E_{h1} - F_v)/k_B T}} \)
For one occupied conduction and valence level, redefine:

\[ n_c \equiv \frac{m_e^* k_B T}{\pi \hbar^2 L_z} \sum_{n=1}^{\infty} e^{(E_{e1}-E_{en})/k_BT} \quad \text{and} \quad n_v \equiv \frac{m_h^* k_B T}{\pi \hbar^2 L_z} \sum_{n=1}^{\infty} e^{(E_{h1}-E_{hn})/k_BT} \]

Then, we can approximate the occupation probabilities:

\[ f_c(\hbar \omega = E_{h1}) \approx 1 - e^{-n_c/n_c} \]
\[ f_v(\hbar \omega = E_{h1}) \approx e^{-p/n_v} \]

The peak gain as a function of \( n \) is:

\[ g_p = g_{\max} \left( f_c - f_v \right) = g_{\max} \left[ 1 - e^{-n_c/n_c} - e^{-p/n_v} \right] \]

Define \( R \equiv \frac{m_h^*}{m_e} \approx \frac{n_v}{n_c} \), then we get:

\[ g_p = g_{\max} \left[ 1 - e^{-n_c/n_c} - e^{-p/(Rn_c)} \right] \]

Transparency occurs where \( g_p = 0 \):

\[ e^{-n_c/n_c} + e^{-p/(Rn_c)} = 1 \]
Fermi Levels and Gain Versus n

- A plot of $f_c$ and $f_v$ versus $n$ can be used to find the transparency current density

\[ g_p = g_{\text{max}} (f_c - f_v) \]

\[ \text{If } f_c = f_v \text{ then } g_p = 0 \]

- Peak gain versus carrier concentration
The total current density can be expressed:

\[ J = J_{rad} + J_{Aug} + J_{leak} \]

for a carrier concentration \( n \):

\[ J_{rad} = qL_z R_{sp} (n) \quad \Rightarrow \quad R_{sp} (n) \approx B n^2 \]

\[ J_{Aug} = qL_z R_{Aug} (n) \quad \Rightarrow \quad R_{Aug} (n) \approx C n^3 \]

A common empirical formula for the peak gain as a function of current density is given by:

\[ g_p (J) = g_0 \left[ 1 + \ln \frac{J}{J_0} \right] = g_0 \ln \frac{J}{J_{tr}} \]

Note: the QW is transparent at \( J = J_{tr} = e^{-1} \cdot J_0 \).

Note: \( J = \eta J_{applied} \) where \( \eta \equiv \) injection efficiency or fraction of applied current captured by the QW. So, \( J_{applied} = J_0 / \eta e \).

From the Auger and radiative terms, \( J \propto n^\beta \), where \( \beta \) is typically between 2 and 3:

\[ g_p (n) = g_0 \left[ 1 + \ln \frac{n^\beta}{n_0^\beta} \right] = g_0 \left[ 1 + \beta \ln \frac{n}{n_0} \right] = g_0 \beta \left[ \frac{1}{\beta} + \ln \frac{n}{n_0} \right] = g_0 \beta \ln \frac{n}{n_{tr}} \]
Scaling Laws: Gain for SQW and MQW Structures

Empirical Formula for Peak Gain - Current Density Relation:

\[ g_w = g_0 \left[ \ln \left( \frac{J_w}{J_0} \right) + 1 \right] \]

where

\[ J_w = \eta J_{\text{applied}} \equiv \text{injected current density for a SQW (single quantum well)} \]

\[ g_w \equiv \text{peak gain coefficient for a SQW, } g_w \propto L_z^{-1} \]

Transparency Current Density:

\[ J_w = J_{\text{tr}} = J_0 e^{-1} \]

For a MQW Structure, the Modal Gain at Threshold is:

\[ G_{\text{th}} = n_w \Gamma_w g_w = \alpha_{\text{tot}} = \alpha_i + \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right) \]

\[ n_w : \# \text{ of wells; } \Gamma_w = \Gamma_{\text{op}} \frac{L_z}{W_{\text{mode}}} : \text{confinement factor per well} \]

General Expression, Modal Gain for a SQW:

\[ G = \Gamma_w g_w = \Gamma_w g_{\text{max}} \left[ f_c \left( \frac{\hbar \omega}{E_{h1}^e (0)} \right) - f_v \left( \frac{\hbar \omega}{E_{h1}^e (0)} \right) \right] = \frac{\Gamma_{\text{op}} L_z}{W_{\text{mode}}} g_{\text{max}} \]

Note: \[ g_{\text{max}} = C_0 \left| \hat{e} \cdot \mathbf{M}_{\text{ch}} \right|^2 \frac{m_r^*}{\pi \hbar^2 L_z} \delta_{nm} \] and for a MQW: \[ G = n_w \Gamma_w g_w \]
Threshold Current Density

Injected Current Density per QW at Threshold: \( J_w = \frac{\eta J_{th}}{n_w} \)

Each QW has gain: \( g_0 \left[ \ln \left( \frac{J_w}{J_0} \right) + 1 \right] \), so **Peak Gain for MQW**:

\[
\frac{n_w g_w}{n_w J_0} = n_w g_0 \left[ \ln \left( \frac{J_w}{J_0} \right) + 1 \right] = n_w g_0 \ln \left( \frac{n_w J_w}{n_w J_0} \right) + 1
\]

**Relationship Between \( J_{th} \) and Cavity Length**:

\[
\left( \frac{n_w J_w}{n_w J_0} \right) = e^{(n_w g_0 - 1)} \quad \Rightarrow \quad \eta J_{th} = n_w J_w = n_w J_0 \exp \left[ \left( \frac{g_w}{g_0} \right) - 1 \right]
\]

\[
J_{th} = \frac{n_w J_0}{\eta} \exp \left[ \left( \frac{g_w}{g_0} \right) - 1 \right]. \quad \text{Note: } n_w \Gamma_w g_w = \alpha_{tot} \quad \text{so:}
\]

**Gain = Loss**

\[
J_{th} = \left( \frac{n_w J_0}{\eta} \right) \exp \left[ \frac{\alpha_{tot}}{n_w \Gamma_w g_0} - 1 \right]
\]

\[
\ln J_{th} = \ln \left( \frac{n_w J_0}{\eta} \right) + \frac{1}{n_w \Gamma_w g_0} \left( \alpha_i + \frac{1}{2L} \ln \frac{1}{R_1 R_2} \right) - 1
\]

- Offset mostly increases with \( n_w \)
- Slope proportional to \( 1/n_w \)

\( n_w J_w = \text{total injected current} \)
\( n_w J_0 e^{-1} = \text{total injected current for transparency} \)
Cavity Length to Minimize Threshold Current

\[
\ln J_{th} = \ln \left( \frac{n_w J_0}{\eta} \right) + \frac{1}{n_w \Gamma_w g_0} \left( \alpha_i + \frac{1}{2L} \ln \frac{1}{R_1 R_2} \right) - 1
\]

\[
= \ln \left( \frac{n_w J_0}{\eta} \right) + \frac{\alpha_i}{n_w \Gamma_w g_0} + \frac{L_{opt}}{L} - 1
\]

Note: \( \ln J_{th} \) varies linearly with \( L^{-1} \) and there is an optimal cavity length \( L_{opt} \) that minimizes the threshold current:

\[
L_{opt} \equiv \frac{1}{2} \frac{1}{n_w \Gamma_w g_0} \ln \left( \frac{1}{R_1 R_2} \right)
\]

**Threshold Current from Threshold Current Density:**

\[
I_{th} = \frac{w L n_w J_0}{\eta} \exp \left[ \frac{\alpha_i}{n_w \Gamma_w g_0} + \frac{L_{opt}}{L} - 1 \right] = \text{const} \cdot L e^{\frac{L_{opt}}{L}}
\]

**Optimum Cavity Length and Minimum Threshold Current:**

Using \( \frac{\partial}{\partial L} I_{th} = 0 = I_{th} \left( \frac{1}{L} - \frac{L_{opt}}{L^2} \right) \) shows \( L = L_{opt} \) minimizes \( I_{th} \).

\[
I_{th}^{\text{min}} = \frac{w L_{opt} n_w J_0}{\eta} \exp \left[ \frac{\alpha_i}{n_w \Gamma_w g_0} \right]
\]
You can't simultaneously optimize the cavity length and the number of wells to minimize $I_{th}$ except for the non-physical condition $\alpha_i = 0$, $\alpha_m = \text{integer} \cdot \Gamma_w g_0$. 

$$I_{th} = \frac{wL \eta N_w J_0}{\eta} \exp \left[ \frac{\alpha_{tot}}{n_w \Gamma_w g_0} - 1 \right] = \text{const} \cdot n_w e^{\frac{\alpha_{tot}}{n_w \Gamma_w g_0}}$$

Optimum # of QWs and Minimum Threshold Current:

Define: $n_{opt} = \frac{\alpha_{tot}}{\Gamma_w g_0}$ so that $I_{th} = \text{const} \cdot n_w e^{\frac{n_{opt}}{n_w}}$

Similar calculation: $\frac{\partial}{\partial n_w} I_{th} = 0 = I_{th} \left( \frac{1}{n_w} - \frac{n_{opt}}{n_w^2} \right)$:

gives us minimum threshold current for $n_w = n_{opt}$

$$n_{opt} = \frac{\alpha_{tot}}{\Gamma_w g_0} = \frac{1}{\Gamma_w g_0} \left[ \alpha_i + \frac{1}{2L} \ln \frac{1}{R_1 R_2} \right]$$

But, in practice, $n_w$ is an integer.

Assumes no coupling effects between well.

Note: $I_{th}^{\min} = \frac{wL n_{opt} J_0}{\eta}$

Can't simultaneously optimize:

If $L = L_{opt}$, then

$$n_w \equiv \frac{\alpha_m}{\Gamma_w g_0} \neq n_{opt}$$

unless $\alpha_i = 0$ (non-physical).
Representative Data

(a) 

Threshold current density (A/cm²) vs. \( L_c (\mu m) \)

- GaAs(7 nm)/Al\(_{0.22}\)Ga\(_{0.78}\)As (5 nm)

\[ \frac{1}{L_c} \ell n R^{-1} (cm^{-1}) \]

(b) 

Threshold current (mA) vs. Cavity length (\( \mu m \))

- Stripe width ~ 4 \( \mu m \)

\[ n_w = 3, 2, 1 \]

\( n\)-GaAs
Strain Effects: Band Structure and Quantum Well Laser Effects
• Biaxial Compression (Compressive Strain): Strained material has a larger lattice constant, resulting in compression in the plane of the wafer and tension in the direction perpendicular to the surface.

• Biaxial Tension (Tensile Strain): Strained material has a smaller lattice constant, resulting in tension in the plane of the wafer and compression in the direction perpendicular to the surface.

• Critical layer thickness: Thickness beyond which dislocations form to accommodate mismatch.
A Few Key Points about Strain Effects on Band Structure

- **Section 4.5**
- Strain modifies the band structure of the valence band, in some cases significantly
  - Band energy and carrier effective mass can both change (notably, \( \text{hh} \) becomes lighter and \( \text{lh} \) becomes heavier in \( k_x \))
  - Degeneracy of \( \text{hh}/\text{lh} \) bands broken
- Strain can change the laser threshold current
- Strain can change the polarization of the emitted light (or polarization dependent gain for an amplifier)
- Reduction in threshold carrier density can reduce non-radiative processes such as Auger recombination
- \( \text{InGaAs/InGaAsP/InAlAs/InP} \)
- \( \text{InGaAs/AlGaAs/GaAs} \)
- \( \text{InGaAs/InGaAsP/GaAs} \)
Assignments
Assignments

• Reading
  – Physics of Photonic Devices (S.L. Chuang)
    • Thurs 2/21: § 10.3 in Chuang and § 8.2.5 in C&C
    • Tues 2/26: § ’s 4.5 & 10.4
    • Thurs 2/28: § ’s 4.5 & 10.4
    • Tues 2/5: § ’s 12.1, 12.2, posted notes on TL
    • Thurs 3/7: § ’s 12.1, 12.2
    • Tues 3/12: § ’s 8.5, 11.1
    • Thurs 3/14: § ’s 11.2
  – Diode Lasers and Photonic Integrated Circuits (Coldren & Corzine)
    • § 8.2.5
    • Appendix 1, 2, 3, 9 (Supplemental)
• Finalize Partner Selection and Preference Sheet
Topics for Next Lecture
Agenda for Thursday, 3/7

• Transfer Matrix Method
• Distributed Feedback Structures
Thank You for Listening!
Course Purpose & Objectives
Course Purpose

- Cover the theory and design of semiconductor devices used in optical communication systems and electronic-photonic integrated circuits
Course Objectives

• Discuss, at a graduate level, key topics in semiconductor physics
• Discuss, at a graduate level, key topics in electromagnetics as applied to photonic devices
• Provide an understanding of active photonic devices used in optical communication systems and photonic integrated circuits
• Provide an understanding of passive photonic devices used in optical communication systems and photonic integrated circuits
Overlap With ECE/PHYS Courses

- Quantum Mechanics (PHYS 486/487)
- Semiconductor Physics (ECE 488)
- E&M (ECE 452/520)

ECE 536
Course Schedule
**Tentative Schedule [1]**

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**Subject to Change**
# Tentative Schedule [3]

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Grading, Expectations, and Policies
Determination of Grade

• Homework: 20%
• Exam I: 25%
• Exam II: 25%
• Class Participation &
  Term Project Presentation: 15%
• Term Project Report: 15%
Homework

• Assigned per posted class schedule on Thursdays, due 1 week later

• TA is Fu-Chen Hsiao, 3211 MNTL
  – Office hours Wednesday from 10-11am, 4034 ECEB
  – TA: good first contact for questions on homework

• Do not copy solutions from others in class or from other sources
Term Project

• Collaborative presentation: teams of 2
• Individual paper
• Details and a list of topics will be provided in early February
Expectations

• Diligence
  – Attend class & participate

• Honesty
  – No cheating on exams or homework
  – Original work on term project
  – Accurate/legitimate representation of any issues affecting homework/exams/project

• Mutual Respect

• Maturity
  – Graduate-level class
Policies

• Where applicable, general university policies on academic affairs will be used

• Any issues involving homework, exams, semester project, etc. should be disclosed and discussed as soon as the issue is known
Text Errata
Text Errata

• Text errata provided by Professor Chuang will be posted on the class website
  – I will include additional errata in the lecture slides

• Handout: inside book cover

• Additional errata:
  – Equation 3.2.18 “=“ should be “>”
    • $|z| > \frac{L}{2}$
  – Equation 3.2.23 “=“ should be “<“
    • $|z| < \frac{L}{2}$
  – Equation 3.2.23: “L/2” not “L2” in exponent
1) Equation 1.3.1 should be:
\[ a(A_x B_{1-x} C) = xa(AC) + (1 - x)a(BC) \]
where \( a(AC) \) is the lattice constant of the binary compound AC and...

2) Equation 1.3.2 should be:
\[ E_g(A_x B_{1-x} C) = xE_g(AC) + (1 - x)E_g(BC) - bx(1 - x) \]

3) Other printing errors in book - see errata posted on website
• Equation 3.6.15 should be:

\[ a_m^{(0)}(t = 0) = 0 \quad \text{not} \quad a_m^{(0)}(t) = 0 \]
Correction: Typo in Book

• Pg. 41, top of page, between equations 2.3.3 and 2.3.4
• Current Text is: “0 = Bn_o p_o = e_r”
• Should Be: “0 = Bn_o p_o - e_r”

The consequence of this is that $Bn_o p_o = e_r$ in equilibrium where there is no optical generation or electrical injection of carriers.
Threshold Current Density

Injected Current Density per QW at Threshold: \( I_w = \frac{\eta J_{th}}{n_w} \)

Each QW has gain: \( g_0 \left[ \ln \left( \frac{I_w}{I_0} \right) + 1 \right] \), so **Peak Gain for MQW**: 

\[
n_w g_w = n_w g_0 \left[ \ln \left( \frac{I_w}{I_0} \right) + 1 \right] = n_w g_0 \left[ \ln \left( \frac{n_w J_w}{n_w J_0} \right) + 1 \right]
\]

Relationship Between \( J_{th} \) and Cavity Length:

\{ **Typo in book** - p 444, should be: substitute 10.3.29 into 10.3.30\}

\[
\left( \frac{n_w J_w}{n_w J_0} \right) = e^{\left( \frac{n_w g_w}{n_w g_0} \right) - 1} \Rightarrow \eta J_{th} = n_w J_w = n_w J_0 \exp \left[ \left( \frac{g_w}{g_0} \right) - 1 \right]
\]

\[
J_{th} = \frac{n_w J_0}{\eta} \exp \left[ \left( \frac{g_w}{g_0} \right) - 1 \right]. \quad \text{Note: } n_w \Gamma_w g_w = \alpha_{tot} \text{ so:}
\]

\[
\text{Gain} = \text{Loss}
\]

\[
J_{th} = \left( \frac{n_w J_0}{\eta} \right) \exp \left[ \frac{\alpha_{tot}}{n_w \Gamma_w g_0} - 1 \right]
\]

\[
\ln J_{th} = \ln \left( \frac{n_w J_0}{\eta} \right) + \frac{1}{n_w \Gamma_w g_0} \left( \alpha_i + \frac{1}{2L} \ln \frac{1}{R_1 R_2} \right) - 1
\]

- Offset mostly increases with \( n_w \)
- Slope proportional to \( 1/n_w \)

\[ n_w J_w = \text{total injected current} \]
\[ n_w J_0 e^{-1} = \text{total injected current for transparency} \]
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