Homework Assignment 4  
Due Date: Wednesday, March 16 (in class)

Reading: Sections 3.4, 4.1-4.7 of text.

1. Using the joint characteristic function show that if $X_1, X_2, X_3, X_4$ are jointly Gaussian (correlated) random variables with zero mean, then

$E[X_1X_2X_3X_4] = E[X_1X_2]E[X_3X_4] + E[X_1X_3]E[X_2X_4] + E[X_1X_4]E[X_2X_3]$

2. Suppose

$\begin{bmatrix} X \\ Y \end{bmatrix} \sim \mathcal{N}\left( \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 & -1 \\ -1 & 1 \end{bmatrix} \right)$

(a) Find $P\{X + Y > 4\}$ in terms of the $Q(\cdot)$ function.
(b) Find $f_{X|Y}(x|y)$.
(c) Find $E[X^2|Y]$.
(d) Find $P\{X \geq 2\}|Y = 2\}$ in terms of the $Q(\cdot)$ function.
(e) Find $\hat{E}[X^2|Y]$.

3. Problem 4.3 of the text.

4. Problem 4.5 of the text. You may skip part (b) of this question.

5. Problem 4.7 of the text.

6. Problem 4.11 of the text. You may skip part (c) of this question.

7. True or False. Determine if the following statements are True or False. You need to justify your answer clearly to get credit – provide a short proof if you say the statement is True, and a counter-example if you say the statement is False.

(a) Let $\Theta \sim \text{Unif}[0, 2\pi]$ and let the random process $(X_t: t \geq 0)$ be defined by $X_t = \cos(2\pi t + \Theta)$. Then $(X_t)$ is an independent increment process.
(b) If $(X_t)$ and $(Y_t)$ are independent WSS Gaussian processes, then the process $(Z_t)$ defined by $Z_t = X_tY_t$ is also WSS.
(c) If $(X_t, t \geq 0)$ is a martingale, then $(X_t)$ is necessarily WSS as well.

8. Problem 4.15 of the text.