ECE 534: SPRING 2017
PREPARATION QUIZ
ISSUED: FEBRUARY 6TH.

This is a self-examination quiz, and it is designed to help you evaluate your background in elementary probability and your understanding of the course material covered in the last three weeks. The first five questions are worth 10 pts each, the sixth and seventh are worth 15 pts, while the last two are worth 20 pts. When writing your solution, please include as much detail as needed to assess the accuracy of your reasoning.

You will not be graded for this work. Make sure to team up with $1-2$ of your colleagues and jointly go through the solutions that will be posted on Tuesday.

- **Problem 1 - Set operations.** Let A, B, C be three events (sets) in the same σ-algebra of some probability space. Write the expression for the following events:
  i) Exactly two events occur.
  ii) Both A and B, but not C, occur.
  iii) If A, B and C are statistically independent events, find the probability of event ii) expressed in terms of the probabilities of A, B, and C.
  iv) Give an exact formula for $P(A \cup B \cup C)$.

- **Problem 2 - σ-algebras.** Let $(\Omega, \mathcal{F}, P)$ be a probability space with $\Omega = \{1, 2, 3\}$. Let $\mathcal{F}$ be the smallest σ-algebra that contains the events $A = \{1\}$ and $B = \{1, 2\}$. Specify the whole list of elements of the σ-algebra. Can you assign a probability to the event $\{2\}$? Justify your answer.

- **Problem 3 - Bayes formula.** You are interested in finding out a patient's probability of having lung cancer if they smoke. Past data tells you that 5% of patients entering your clinic have lung cancer; 10% of the clinic's patients are smokers. You might also know that among those patients diagnosed with lung cancer, 38% are smokers. If the patient is a smoker, what is his probability of having lung cancer?

- **Problem 4 - Random variables.** Define the notion of a RV. Define the CDF of a RV, and specify the three characterizing properties of a CDF.

- **Problem 5 - Expectation and variance.** Find the expectation of a random variable X with a geometric distribution $Geom(p)$, $0 < p < 1$, that has probability mass function $P\{X = k\} = (1-p)^{k-1}p$, $k \in \{1, 2, \ldots\}$.

- **Problem 6 - Functions of random variables.** Let X be uniformly distributed over $(0, 1)$. Find the CDF and pdf of the RV Y defined as $Y = X^n$.

- **Problem 7 - Jointly distributed random variables.** Let X and Y be two RVs over the same probability space. Define the joint CDF of X and Y, $F_{X,Y}(x,y)$, and show that $\lim_{x,y\to\infty} F_{X,Y}(x,y) = 1$. Make sure you properly use continuity of probability arguments.

- **Problem 8 - Bonus problem.** Consider a random chord of a circle. What is the probability that the length of the chord will be greater than the side of the equilateral triangle inscribed in that circle?