ECE 534: SPRING 2017
HOMEWORK I
ISSUED: 5TH OF SEPTEMBER. DUE SEPTEMBER 18TH.

Note: The HW is due in class, not later than 5 min into the class time.

• **Problem 1.** Prove that the σ-algebra generated by a collection of subsets \( I = \{I_1, \ldots, I_n, \ldots\} \) of \( \Omega \) equals the intersection of all σ-algebras containing \( I \). Let \( \Sigma = \{1, 2, 3, 4\} \). Find the smallest σ-algebra containing \( I_1 = \{1, 3\} \). Find the smallest σ-algebra containing \( I_1 = \{1\} \) and \( I_2 = \{3\} \).

• **Problem 2.** Prove that if \( X \) is a \((\Omega, F)\)-measurable, then \( X^2 \) is \((\Omega, F)\)-measurable as well. In other words, if \( X \) is a random variable over some measurable space, so is \( X^2 \). Also, show that the only random variables over a measurable space \((\Omega, F)\) for which \( F = (\emptyset, \Omega) \) are constant functions.

• **Problem 3.** Find an example that illustrates that \(|X|\) may be a random variable, while \(X\) itself may not be a valid random variable.

• **Problem 4.** Verify that \( P(\{n\}) = 2^{-n} \) represents a valid probability law on a certain probability space for which the sample space is \( \Omega = \mathbb{N} \) (the natural numbers). Make sure to first specify the probability space.

• **Problem 5.** When coin \( a \) is flipped, it comes up heads with probability 1/4, whereas when coin \( b \) is flipped it comes up heads with probability 3/4. Suppose that one of these coins is randomly chosen and flipped twice. If both flips land heads, what is the probability that coin \( b \) was the one flipped?

• **Problem 6.** An infinite sequence of independent trials is to be performed. Each trial results in success with probability \( p \) and failure with probability \( 1 - p \). What is the probability that a) At least one success occurs in the first \( n \) trials? b) Exactly \( k \) successes occur in the first \( n \) trials, where \( 0 \leq k \leq n \)? Make sure to define the probability space and carefully justify all your answers.

• **Problem 7.** Problems 1.13, 1.17, 1.19, 1.21 from Prof. Hajek’s text.