

**ECE 534: SPRING 2017**  
**HOMEWORK I**  
**ISSUED: JANUARY 27TH. DUE FEBRUARY 8TH.**

- **Problem 1.** Let  $\Sigma = \{1, 2, 3, 4\}$ . Find the smallest  $\sigma$ -algebra containing  $A = \{1, 3\}$ . Find the smallest  $\sigma$ -algebra containing  $A = \{1\}$  and  $B = \{3\}$ .
- **Problem 2.** Verify that  $P(\{n\}) = 2^{-n}$  is a valid probability law on a certain probability space for which the sample space is  $\Omega = \mathbb{N}$  (the natural numbers). Make sure that you specify the probability space.
- **Problem 3.** When coin a is flipped, it comes up heads with probability  $1/4$ , whereas when coin b is flipped it comes up heads with probability  $3/4$ . Suppose that one of these coins is randomly chosen and flipped twice. If both flips land heads, what is the probability that coin b was the one flipped?
- **Problem 4.** An infinite sequence of independent trials is to be performed. Each trial results in success with probability  $p$  and failure with probability  $1 - p$ . What is the probability that a) At least one success occurs in the first  $n$  trials? b) Exactly  $k$  successes occur in the first  $n$  trials, where  $0 \leq k \leq n$ ? c) All trials are successful. Make sure to define the probability space and carefully justify all your answers.
- **Problem 5.** Problems 1.13, 1.17, 1.19, 1.21.