# Homework Set 4 - Solutions

## ECE 530

October 19, 2015

## 1

Apply the Gauss-Seidel iteration to the system

$$A = \begin{bmatrix} 0.96326 & 0.81321 \\ 0.81321 & 0.68654 \end{bmatrix} \qquad b = \begin{bmatrix} 0.88824 \\ 0.74988 \end{bmatrix}$$

Use $x^{(0)} = [0.331160.70000]^T$ and explain what happens

$$L + D = \begin{bmatrix} 0.96326 & 0 \\ 0.81321 & 0.68654 \end{bmatrix} \qquad U = \begin{bmatrix} 0 & 0.81321 \\ 0 & 0 \end{bmatrix}$$

$$G_{GS} = -(L+D)^{-1}U = \begin{bmatrix} 0 & -0.84423 \\ 0 & 0.99999 \end{bmatrix} \qquad c = (L+D)^{-1}b = \begin{bmatrix} 0.92212 \\ 0.00000568 \end{bmatrix}$$

$$x^{(i+1)} = -(L+D)^{-1}Ux^{(i)} + (L+D)^{-1}b = \begin{bmatrix} 0.3311598 \\ 0.6999993 \end{bmatrix}$$

The fact that $\Delta x$ between $x^{(1)}$ and $x^{(0)}$ is very small is a warning that the Gauss-Seidel method may not converge or may converge slowly. Depending on the $\epsilon$ you choose, the number of iterations could be in the thousands, if not hundreds of thousands.

A closer look at the convergence of this stationary method: find the eigenvalues of $G$.
"A necessary and sufficient condition for the convergence is that the magnitude of matrix $G$ is smaller than 1 (or lies withing the unit circle in the complex plane)"

$$\text{eig}(G) = \begin{bmatrix} 0 \\ 0.9999909 \end{bmatrix}$$

The system does converge, but extremely slowly.

## 2

Solve problem 1 using the conjugate gradient method. What happens now?

```
A = [0.96326, 0.81321;
    0.81321, 0.68654];
b = [0.88824;
    0.74988];

i = 0;
x = zeros(size(b));
%x = [0.33116; 0.70000];
r_o = b - A * x;
r_n = r_o;
epsi = 1e-7; %1e-6
while(norm(r_n, inf))>epsi
    if i==0
        d = r_n;
    else
        b = (r_n'*r_n)/(r_o'*r_o);
        d = r_n + b * d;
    end
    a = (d'*r_n)/(d'*A*d);
    x = x + a * d;
    r_o = r_n;
    r_n = r_o - a * A * d;
    i = i + 1;
end

x
i

x =
   0.394727656563906
   0.624702890464605
i =
     2
```

Note: It doesn't matter whether you start with $x = [0,0]^T$ or $x^{(0)}$ from Problem 1
Note: If $\epsilon = 1e^{-6}$, the solution will prematurely stop after 1 iteration.

# 3

For the two-bus case in Problem 3 of Problem Set 1, place a transformer with p.u. reactance $X = 0.2j$ an the tap $t = 1\langle 5\,\text{deg}$ between Bus 2 and the load.
Find the new admittance matrix.

$$Ybus = \begin{bmatrix} -j9.9 & j10 & 0 \\ j10 & -j14.9 & -0.436 + j4.981 \\ 0 & 0.436 + j4.981 & -j5 \end{bmatrix}$$

$$I_1 = \frac{V_1 - V_2}{j0.1} + V_1(j0.1) = (-j9.9)V_1 + (j10)V2$$

$$I_2 = \frac{V_2 - V_1}{j0.1} + V_2(j0.1) + V_2(\frac{-j5}{1^2}) + V_3(\frac{j5}{e^{-j5}}) = (j10)V_1 + (-j14.9)V_2 + (-0.436 + j4.981)V_3$$

$$I_3 = V_2(\frac{j5}{e^{j5}}) + V_3(\frac{-j5}{1}) = (0.436 + j4.981)V_2 + (-j5)V_3$$

There should be 16 entries in the Jacobian. We take the partial derivative of $P_2$, $Q_2$, $P_3$, and $Q_3$ with respect to $V_2$, $V_3$, $\theta_2$ and $\theta_3$.

# 4

For the five-bus case in Problem Set 2, solve it again using the dc power flow model. Compare the solution with the one you obtained using the N-R power flow method.

## Set Up (from previous)

```
Ybus = [3.7290-j*49.7203    0                    0                    0
        0                   2.6783-j*28.4590     0                    -0.8928+j*9.9197
        0                   0                    7.4580-j*99.4406     -7.4580+j*99.4406
        0                   -0.8928+j*9.9197     -7.4580+j*99.4406    11.9219-j*147.9589
        -3.7290+j*49.7203   -1.7855+j*19.8393        0                -3.5711+j*39.6786

 PG = [0 0 5.2 0 0];
 PD = [0 8 .8 0 0];
```

## DC Power Flow

```
 B = imag(Ybus(2:5,2:5));
 P = (PG(2:5)-PD(2:5))';

 % in radians
 THETA = -inv(B)*P;
 THETAdeg = rad2deg(THETA);
```

| N-R V | DC V | N-R $\theta(rad)$ | N-R $\theta(rad)$ |
|---|---|---|---|
| 0.8338 | 1 | -0.3910 | -0.3574 |
| 1.05 | 1 | -0.0104 | -0.0068 |
| 1.0193 | 1 | -0.0494 | -0.0511 |
| 0.9743 | 1 | -0.0794 | -0.0840 |