Apply the Gauss-Seidel iteration to the system

\[
A = \begin{bmatrix}
0.96326 & 0.81321 \\
0.81321 & 0.68654
\end{bmatrix} \quad b = \begin{bmatrix}
0.88824 \\
0.74988
\end{bmatrix}
\]

Use \( x^{(0)} = [0.33116\quad 0.70000]^T \) and explain what happens

\[
L + D = \begin{bmatrix}
0.96326 & 0 \\
0.81321 & 0.68654
\end{bmatrix} \quad U = \begin{bmatrix}
0 & 0.81321 \\
0 & 0
\end{bmatrix}
\]

\[
G_{GS} = -(L + D)^{-1}U = \begin{bmatrix}
0 & -0.84423 \\
0 & 0.99999
\end{bmatrix} \quad c = (L + D)^{-1}b = \begin{bmatrix}
0.92212 \\
0.00000568
\end{bmatrix}
\]

\[
x^{(i+1)} = -(L + D)^{-1}Ux^{(i)} + (L + D)^{-1}b = \begin{bmatrix}
0.3311598 \\
0.6999993
\end{bmatrix}
\]

The fact that \( \Delta x \) between \( x^{(1)} \) and \( x^{(0)} \) is very small is a warning that the Gauss-Seidel method may not converge or may converge slowly. Depending on the \( \epsilon \) you choose, the number of iterations could be in the thousands, if not hundreds of thousands.

A closer look at the convergence of this stationary method: find the eigenvalues of \( G \).

"A necessary and sufficient condition for the convergence is that the magnitude of matrix \( G \) is smaller than 1 (or lies withing the unit circle in the complex plane)"

\[
eig(G) = \begin{bmatrix}
0 \\
0.9999909
\end{bmatrix}
\]

The system does converge, but extremely slowly.
Solve problem 1 using the conjugate gradient method. What happens now?

\[ A = \begin{bmatrix} 0.96326, & 0.81321; \\ 0.81321, & 0.68654 \end{bmatrix}; \]
\[ b = \begin{bmatrix} 0.88824; \\ 0.74988 \end{bmatrix}; \]
\[ i = 0; \]
\[ x = \text{zeros(size(b))}; \]
\[ x = \begin{bmatrix} 0.33116; \\ 0.70000 \end{bmatrix}; \]
\[ r_o = b - A * x; \]
\[ r_n = r_o; \]
\[ \text{epsi} = 1e-7; \]
\[ \text{while} (\text{norm}(r_n, \infty) > \text{epsi}) \]
\[ \text{if } i = 0 \]
\[ d = r_n; \]
\[ \text{else} \]
\[ b = (r_n' * r_n) / (r_o' * r_o); \]
\[ d = r_n + b * d; \]
\[ \text{end} \]
\[ a = (d' * r_n) / (d' * A * d); \]
\[ x = x + a * d; \]
\[ r_o = r_n; \]
\[ r_n = r_o - a * A * d; \]
\[ i = i + 1; \]
\[ \text{end} \]
\[ x = \begin{bmatrix} 0.394727656563906; \\ 0.624702890464605 \end{bmatrix}; \]
\[ i = 2 \]

Note: It doesn't matter whether you start with \( x = [0, 0]^T \) or \( x^{(0)} \) from Problem 1

Note: If \( \epsilon = 1e^{-6} \), the solution will prematurely stop after 1 iteration.
For the two-bus case in Problem 3 of Problem Set 1, place a transformer with p.u. reactance \( X = 0.2j \) an the tap \( t = 15\text{deg} \) between Bus 2 and the load. Find the new admittance matrix.

\[
Y_{bus} = \begin{bmatrix}
-j9.9 & j10 & 0 \\
-j10 & -j14.9 & -0.436 + j4.981 \\
0 & 0.436 + j4.981 & -j5
\end{bmatrix}
\]

\[
I_1 = \frac{V_2 - V_1}{j0.1} + V_1(j0.1) = (-j9.9)V_1 + (j10)V_2
\]

\[
I_2 = \frac{V_2 - V_1}{j0.1} + V_2(j0.1) + V_2(\frac{j5}{e^{j5}}) + V_3(\frac{j5}{e^{j5}}) = (j10)V_1 + (-j14.9)V_2 + (-0.436 + j4.981)V_3
\]

\[
I_3 = V_2(\frac{j5}{e^{j5}}) + V_3(\frac{j5}{e^{j5}}) = (0.436 + j4.981)V_2 + (-j5)V_3
\]

There should be 16 entries in the Jacobian. We take the partial derivative of \( P_2 \), \( Q_2 \), \( P_3 \), and \( Q_3 \) with respect to \( V_2 \), \( V_3 \), \( \theta_2 \) and \( \theta_3 \).
For the five-bus case in Problem Set 2, solve it again using the dc power flow model. Compare the solution with the one you obtained using the N-R power flow method.

**SET UP (FROM PREVIOUS)**

\[
Y_{bus} = \begin{bmatrix}
3.7290-j*49.7203 & 0 & 0 & -3.7290+j*49.7203 \\
0 & 2.6783-j*28.4590 & 0 & -0.8928+j*9.9197 \\
0 & 0 & 7.4580-j*99.4406 & -7.4580+j*99.4406 \\
0 & -0.8928+j*9.9197 & -7.4580+j*99.4406 & 11.9219-j*147.9589 \\
-3.7290+j*49.7203 & -1.7855+j*19.8393 & 0 & -3.5711+j*39.6786
\end{bmatrix}
\]

\[
PG = \begin{bmatrix} 0 & 0 & 5.2 & 0 & 0 \end{bmatrix};
\]

\[
PD = \begin{bmatrix} 0 & 8 & .8 & 0 & 0 \end{bmatrix};
\]

**DC POWER FLOW**

\[
B = \text{imag}(Y_{bus}(2:5,2:5));
\]

\[
P = (PG(2:5)-PD(2:5))';
\]

% in radians
\[
\text{THETA} = -\text{inv}(B)*P;
\]

\[
\text{THETAdeg} = \text{rad2deg}(\text{THETA});
\]

<table>
<thead>
<tr>
<th>N-R V</th>
<th>DC V</th>
<th>N-R $\theta$ (rad)</th>
<th>N-R $\theta$ (rad)</th>
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<tr>
<td>0.8338</td>
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<td>-0.3574</td>
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<tr>
<td>0.9743</td>
<td>1</td>
<td>-0.0794</td>
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