Homework Set 2 - Solutions

ECE 530

September 23, 2015

1

$$
\begin{aligned}
f(x) &= Ax - y, \qquad f(x^*) = 0 \\
f(x^*) &\approx f(x) + J(x)(X^* - x) \\
x^* &= x + J(x)^{-1}(f(x^*) - f(x)) \\
x^* &= x - J(x)^{-1}(f(x)) \\
J(x) &= \frac{d(f(x))}{dx} \\
&= \frac{d(Ax - y)}{dx} \\
&= A
\end{aligned}
$$

$$
\begin{aligned}
x^{(1)} &= x^{(0)} - J(x^{(0)})^{-1} f(x^{(0)}) \\
&= x^{(0)} - A^{-1}(Ax^{(0)} - y) \\
&= x^{(0)} - A^{-1}Ax^{(0)} + A^{-1}y \\
&= x^{(0)} - x^{(0)} + A^{-1}y \\
&= A^{-1}y
\end{aligned}
$$

$$
\begin{aligned}
f(x^{(1)}) &= Ax^{(1)} - y \\
&= AA^{-1}y - y \\
&= 0
\end{aligned}
$$

After one iteration, when we substitute $x^{(1)}$ into $f(x)$ and it goes to 0, it shows the convergence of the Newton-Raphson method in one iteration for a linear system of equations.

2

$$
\begin{aligned}
\frac{\partial f_i^Q}{\partial \theta_i} &= \sum_{k\neq i}^{n} |V_i||V_k|(G_{ik}\cos(\theta_{ik}) + B_{ik}\sin(\theta_{ik})) \\
\frac{\partial f_i^Q}{\partial \theta_j} &= -|V_i||V_j|(G_{ij}\cos(\theta_{ij}) + B_{ij}\sin(\theta_{ij})) \\
\frac{\partial f_i^Q}{\partial |V_i|} &= -2|V_i|B_{ii} + \sum_{k\neq i}^{n} |V_k|(G_{ik}\sin(\theta_{ik}) - B_{ik}\cos(\theta_{ik})) \\
\frac{\partial f_i^Q}{\partial |V_j|} &= |V_i|(G_{ij}\sin(\theta_{ij}) - B_{ij}\cos(\theta_{ij}))
\end{aligned}
$$

3

Eliminate and normalize:

$$b_k^k = \frac{b_k^{k-1}}{a_{kk}^{k-1}}$$

And we know,

$$b_k^k = \frac{b_k - \sum_{m=1}^{k-1} a_{km}^{(m-1)} b_m^{(m)}}{a_{kk}^{(k-1)}} \implies b_k = \sum_{m=1}^{k} a_{km}^{(m-1)} b_m^{(m)}$$

We are given the structure of L and $\hat{b}$ and we want to prove that $L^{-1}b = \hat{b}$, equivalently $L\hat{b} = b$.

$$
\begin{aligned}
L\hat{b} &= \begin{bmatrix} a_{11}^{(0)} & & & 0 \\ a_{21}^{(0)} & a_{22}^{(1)} & & \\ \dots & \dots & & \\ a_{n1}^{(0)} & a_{n2}^{(1)} & \dots & a_{nn}^{(n-1)} \end{bmatrix} \begin{bmatrix} b_1^{(1)} \\ b_2^{(2)} \\ \dots \\ b_n^{(n)} \end{bmatrix} = \begin{bmatrix} a_{11}^{(0)} b_1^{(1)} \\ a_{21}^{(0)} b_1^{(1)} + a_{22}^{(1)} b_2^{(2)} \\ \dots \\ a_{n1}^{(0)} b_1^{(1)} + a_{n2}^{(1)} b_2^{(2)} + \dots + a_{nn}^{(n-1)} b_n^{(n)} \end{bmatrix} \\
&= \begin{bmatrix} \sum_{m=1}^{1} a_{1m}^{(m-1)} b_m^{(m)} \\ \sum_{m=2}^{k} a_{2m}^{(m-1)} b_m^{(m)} \\ \dots \\ \sum_{m=n}^{k} a_{nm}^{(m-1)} b_m^{(m)} \end{bmatrix} = \begin{bmatrix} b_1^{(0)} \\ b_2^{(0)} \\ \dots \\ b_n^{(0)} \end{bmatrix} = b
\end{aligned}
$$

4

```
clear all
```

LU

```
A = [5 0 0 -4; 0 4 0 -3; 0 0 3 -2; -4, -3, -2, 10];
s = length(A);
U = A;
L = zeros(s,s);
for j=1:s,
    L(j,j) = 1;
    for i=(1+j):s
       c = U(i,j)/U(j,j);                %Scaling
       U(i,j:s)=U(i,j:s)-U(j,j:s)*c;
       L(i,j) = c;
    end
end
```

Forward Substitution

```
b = [1; 2; 3; 4];
for i = 2:s
    for j = 1:(i-1)
        b(i) = b(i) - L(i,j)*b(j);
    end
end
y = b;
```

Backward Substitution

```
for i = s:-1:1
    for j = (i+1):s
        b(i) = b(i)-U(i,j)*b(j);
    end
    b(i) = b(i)/U(i,i);
end
x = b
```

```
x =
    2.2642
    2.4352
    2.7202
    2.5803
```

5

SET UP

```
Ybus = [3.73-j*49.72  0             0             0             -3.73+j*49.72;
        0             2.68-j*28.46  0             -0.89+j*9.92  -1.79+j*19.84;
        0             0             7.46-j*99.44  -7.46+j*99.44  0;
        0             -0.89+j*9.92  -7.46+j*99.44 11.92-j*147.96 -3.57+j*39.68;
        -3.73+j*49.72 -1.79+j*19.84 0             -3.57+j*39.68  9.09-j*108.58];

 PG = [0 0 5.2 0 0];
 QG = [0 0 0 0 0];
 PD = [0 8 .8 0 0];
 QD = [0 2.8 .4 0 0];

 syms V2 delta2 V3 delta3 V4 delta4 V5 delta5;
 V = [1 V2 1.05 V4 V5];
 delt = [0 delta2 delta3 delta4 delta5];

 % Create the elements of the power flow equations
 for k = 1:5
  for i = 1:5
   P(i,k) = V(k)*V(i)*(real(Ybus(i,k))*cos(delt(i)-delt(k))
          + imag(Ybus(i,k))*sin(delt(i)-delt(k)));
   Q(i,k) = V(k)*V(i)*(real(Ybus(i,k))*sin(delt(i)-delt(k))
          - imag(Ybus(i,k))*cos(delt(i)-delt(k)));
  end
 end

 % The power flow equations
 PF_P = [P(2,2)+P(2,4)+P(2,5)+PD(2);  P(3,3)+P(3,4)-PG(3)+PD(3);
         P(4,2)+P(4,3)+P(4,4)+P(4,5); P(5,1)+P(5,2)+P(5,4)+P(5,5)];

 PF_Q = [Q(2,2)+Q(2,4)+Q(2,5)+QD(2);  Q(3,3)+Q(3,4)-QG(3)+QD(3);
         Q(4,2)+Q(4,3)+Q(4,4)+Q(4,5); Q(5,1)+Q(5,2)+Q(5,4)+Q(5,5)];

  % Unknowns
  numers = [PF_P(1) PF_P(2) PF_P(3) PF_P(4) PF_Q(1) PF_Q(3) PF_Q(4)];
  denoms = [delta2  delta3  delta4  delta5  V2      V4      V5];

  % f(x) Vector
  fOfX = transpose(numers);

  % Build the Jacobian
```

```
  for m = 1:7
      for n = 1:7
          Jacob(m,n) = diff(numers(m),denoms(n));
      end
  end
```

ITERATE

```
deltaX = [1,1,1,1,1,1,1];
[delt2, delt3, delt4, delt5] = deal(0);
[volt2, volt4, volt5,iter] = deal(1);

toSub = [delta2,delta3,delta4,delta5,V2,V4,V5];
daSub = [delt2,delt3,delt4,delt5,volt2,volt4,volt5];

while (norm(deltaX,inf) > .001)
    f = double(subs(fOfX, toSub, daSub));
    J = double(subs(Jacob,toSub, daSub));

    deltaX = -inv(J)*f;
    x_New = transpose(daSub)+deltaX;
    daSub = transpose(x_New);

    x(:,iter) = x_New;
    n(:,iter) = norm(deltaX,inf);
    iter = iter+1;
end

S_sym   = [P(1,1)+P(1,5);   Q(1,1)+Q(1,5);   Q(3,3)+Q(3,4)+QD(3)];
S_out   = double(subs(S_sym, toSub, daSub));
```

PRINT RESULTS

```
fprintf('\nSOLUTION\n')
fprintf('\t\t\t\t\t  Iterations\n')
fprintf('Variable \t 1 \t\t\t 2 \t\t\t 3 \t\t\t 4 \n')
fprintf('delta2   \t %f \t %f\t%f\t%f \n',x(1,1),x(1,2),x(1,3),x(1,4))
fprintf('delta3   \t %f \t %f \t %f \t %f \n',x(2,1),x(2,2),x(2,3),x(2,4))
fprintf('delta4   \t %f \t %f \t %f \t %f \n',x(3,1),x(3,2),x(3,3),x(3,4))
fprintf('delta5   \t %f \t %f \t %f \t %f \n',x(4,1),x(4,2),x(4,3),x(4,4))
fprintf('V2       \t %f \t %f \t %f \t %f \n',x(5,1),x(5,2),x(5,3),x(5,4))
fprintf('V4       \t %f \t %f \t %f \t %f \n',x(6,1),x(6,2),x(6,3),x(6,4))
fprintf('V5       \t %f \t %f \t %f \t %f \n',x(7,1),x(7,2),x(7,3),x(7,4))
```

```
fprintf('\n\t\t\t\t\t\t\t  Infinity Norm\n')
fprintf('Iteraion \t 1 \t\t\t 2 \t\t\t 3 \t\t\t 4 \n')
fprintf('Inf Norm \t %f \t %f \t %f \t %f \n',n(1),n(2),n(3),n(4))

fprintf('\n\t\t\t\t\t\t\t  Power Output\n')
fprintf('Power\t Bus \t Value (in p.u.)\n')
fprintf('%c \t\t %c \t\t %f \n','P', '1', S_out(1))
fprintf('%c \t\t %c \t\t %f \n','Q', '1', S_out(2))
fprintf('%c \t\t %c \t\t %f \n','Q', '3', S_out(3))
```

```
SOLUTION
                           Iterations
Variable     1            2            3            4
delta2       -0.323072    -0.375463   -0.390254    -0.391043
delta3       0.003512     -0.008496    -0.010334    -0.010410
delta4       -0.038049    -0.047678    -0.049375    -0.049446
delta5       -0.073019    -0.078277    -0.079329    -0.079377
V2           0.942889     0.851800     0.834605     0.833770
V4           1.042280     1.022131     1.019426     1.019298
V5           1.011629     0.979374     0.974527     0.974297

                               Infinity Norm
Iteraion     1            2            3            4
Inf Norm     0.323072     0.091089     0.017195     0.000835

                               Power Output
Power     Bus      Value (in p.u.)
P         1        3.948442
Q         1        1.142329
Q         3        3.375143
```