Homework 2 Solution Sketch

1. K-set consensus: Consider the following algorithm:

   Step 1: Send input to all processes

   Step 2: Wait until values are received from at least n-f processes

   Step 3: Choose output to be the smallest of the values received

   Proof of correctness: Consider k = f+1. Consider the inputs at the n processes. Let v1, v2, …, vk denote the smallest k inputs among these n processes, such that v1<=v2<=…<= vk. Thus, the inputs at the remaining processes are no smaller than vk.

   Since each non-faulty process P waits for at least n-f values at Step 2, it must receive at least one of the smallest k values (since n-f > n-k). Thus, the minimum obtained at Step 3 cannot exceed vk. Since the minimum is obtained over received values, the minimum must then take a value in the set {v1,v2,…,vk}.

2. No, it does not. To prove this, we need to only identify one execution in which consensus is not achieved.

   Consider the case of n = 4f where f processes are faulty. Then n/2+f = 3f. The case when n < 4f can be proved similarly.

   Suppose that the king in the last phase of the algorithm is a faulty process. Suppose that all non-faulty processes begin the last phase with an identical preference, say 0.

   Observe that since there are 3f non-faulty processes, only 3f values received in round 2k-1 with k=f+1 (i.e., last round) are guaranteed to be 0. The remaining f values come from faulty processes, and maybe arbitrary. Thus, although maj = 0 for all non-faulty processes after round 2k-1 with k=f+1, it is possible that for some non-faulty processes mult > 3f = n/2+f and for others mult = 3f = n/2+f. Consider these two types of processes:

   - Non-faulty processes for which maj = 0 and mult > n/2+f. Each such process will set pref as 0 in round k with k=f+1, which then becomes the output of the process, since this is the last phase.

   - Non-faulty processes for which mult = 3f = n/2+f. Each such process will set its pref to equal the value received from the phase king – since phase king in last phase is faulty, the phase king may send value 1 as king-maj in the last round to these processes, causing these processes to set their pref = 1, which becomes their output.

     Thus, agreement is not achieved.