

Please type your answers and submit via Compass2g.

**See the course handout for information regarding the 48-hour extension for homework, and policy on late submissions.**

*In answering any of the questions below, you may use results derived in the class (or assigned reading material) without proof.*

*For questions 2 and 3, you need not present the proofs of the results as part of your homework, but you may want to write the proofs for your own benefit.*

1. (10 points) For Byzantine consensus, in addition to the usual termination and agreement condition, suppose that we require the following validity condition to hold: the decision (i.e., output) must equal the input of a non-faulty process.

Let  $n$  be the number of processes. Let the input of each process be an integer in  $[0, m-1]$ . Thus, the input of each process takes one of the  $m$  possible values.

To be able to achieve Byzantine consensus with the above validity, agreement and termination properties, in presence of up to  $f$  Byzantine faulty processes, prove that  $n \geq f * \max(3, m) + 1$  is *necessary* and *sufficient*.

2. (5 points) The above problem assumes (without stating explicitly) that the communication network consists of a point-to-point links. In such a network, when process  $p_1$  sends a message to process  $p_2$ , no other non-faulty process will hear (i.e., receive) that message.

Suppose that a system instead uses a broadcast channel to connect all the processes. Suppose that when a process sends a message on the broadcast channel, all the processes receive the message.

For this system, state the number of processes necessary and sufficient to achieve Byzantine consensus.

3. (5 points) For a system with the broadcast channel, assume that in each round, each process may send one message on the broadcast channel. To be able to tolerate up to  $f$  **crash** failures, state a tight bound on the number of rounds necessary for achieving consensus. In this case, the validity condition requires that the decision should equal the input of one of the processes.

