UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 498MH Signal and Image Analysis

Homework 7
Fall 2014

Assigned: Thursday, October 23, 2014 Due: Wednesday, November 5, 2014

Reading:
http://www.vyssotski.ch/BasicsOfInstrumentation/SpikeSorting/Design_of_FIR_Filters.pdf

1 Windowed FIR Design

Do **one** of the following three problems.

Problem 7.1.1

Suppose you have a signal, $x_c(t)$, that is sampled at $F_s = 16,000$ samples/second, creating a signal $x[n]$. You would like to implement a discrete time highpass filter with a cutoff frequency of $f_c = 4000$Hz.

(a) What is the discrete-time cutoff frequency, ω_c, in radians/sample?

(b) Define $D(\omega) = 1$ for $|\omega| > \omega_c$, $D(\omega) = 0$ otherwise. What is its inverse DTFT, $d[n]$?

(c) Use windowing to create $h[n]$, a causal approximation to $d[n]$. Suppose that you are willing to tolerate a stopband ripple of -20dB, therefore you are able to use a rectangular window. You want the separation between passband and stopband to be 800Hz, i.e., you want a passband ripple to peak at $f = 4400$Hz, while the first stopband ripple peaks at $f = 3600$Hz. How many nonzero samples does $h[n]$ need to have?

(d) Same as part (c), but now you are only willing to tolerate a -50dB stopband ripple, so you will need to use a Hamming window.

(e) For part (d), write an explicit formula that would allow you to compute the values of every sample $h[n]$, in terms of n. There should be no variables other than n in your answer.

Problem 7.1.2

Suppose you have a signal, $x_c(t)$, that is sampled at $F_s = 16,000$ samples/second, creating a signal $x[n]$. You would like to implement a discrete time bandpass filter with a passband of $f_1 \leq f \leq f_2$, for $f_1 = 2000$Hz, $f_2 = 6000$Hz.

(a) What are the discrete-time cutoff frequencies, ω_1 and ω_2, in radians/sample?

(b) Define $D(\omega) = 1$ for $\omega_1 < |\omega| < \omega_2$, $D(\omega) = 0$ otherwise. What is its inverse DTFT, $d[n]$?

(c) Use windowing to create $h[n]$, a causal approximation to $d[n]$. Suppose that you are willing to tolerate a stopband ripple of -20dB, therefore you are able to use a rectangular window. You want the separation between passband and stopband to be 800Hz, i.e., you want passband ripples that peak at $f = 2400$Hz and $f = 5600$Hz, and you want stopband ripples that peak at $f = 1600$Hz and $f = 6400$Hz. How many nonzero samples does $h[n]$ need to have?

(d) Same as part (c), but now you are only willing to tolerate a -50dB stopband ripple, so you will need to use a Hamming window.

(e) For part (d), write an explicit formula that would allow you to compute the values of every sample $h[n]$, in terms of n. There should be no variables other than n in your answer.

Problem 7.1.3

Suppose you have a signal, $x_c(t)$, that is sampled at $F_s = 16,000$ samples/second, creating a signal $x[n]$. You would like to implement a discrete time stopband filter with stopband of $f_1 < f < f_2$ for $f_1 = 2000$Hz, $f_2 = 6000$Hz.

(a) What are the discrete-time cutoff frequencies, ω_1 and ω_2, in radians/sample?

(b) Define $D(\omega) = 0$ for $\omega_1 < |\omega| < \omega_2$, $D(\omega) = 1$ otherwise. What is its inverse DTFT, $d[n]$?

(c) Use windowing to create $h[n]$, a causal approximation to $d[n]$. Suppose that you are willing to tolerate a stopband ripple of -20dB, therefore you are able to use a rectangular window. You want the separation between passband and stopband to be 800Hz, i.e., you want passband ripples that peak at $f = 1600$Hz and $f = 6400$Hz, and you want stopband ripples that peak at $f = 5600$Hz. How many nonzero samples does $h[n]$ need to have?

(d) Same as part (c), but now you are only willing to tolerate a -50dB stopband ripple, so you will need to use a Hamming window.

(e) For part (d), write an explicit formula that would allow you to compute the values of every sample $h[n]$, in terms of n. There should be no variables other than n in your answer.