UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 498MH Signal and Image Analysis

Homework 4
Fall 2014

Assigned: Thursday, September 25, 2014 Due: Thursday, October 2, 2014

Reading: Mark Hasegawa-Johnson, *Lecture Notes in Speech Production, Speech Coding and Speech Recognition*, Chapter 1: Basics of Digital Signal Processing, `http://isle.illinois.edu/~hasegawa/notes/chap1.pdf`
Announcement: Exam 1, in class on Friday October 3, will cover homeworks 1-4

1 Damped Sinusoids: CTFS, CTFT, DTFT, and DFT

Do **one** of the following three problems.

Problem 4.1.1

The vowel /a/ is characterized by formant frequencies at $F_1 = 900$ and $F_2 = 1100$ Hertz, and with bandwidths of roughly $B_1 = B_2 = 150$ Hertz. This problem will focus only on the positive-frequency part of the first formant ringing, and will ignore amplitude and phase, thus

$$x(t) = e^{-(\pi 150 - j2\pi 900)t}u(t)$$

(a) Find $X(\omega)$ and $|X(\omega)|^2$, the CTFT and its associated power spectrum.

(b) Suppose that $y(t)$ is the periodic repetition of $x(t)$, repeated once every 10ms. Find the Fourier series coefficients Y_k, and the associated power spectrum $|Y_k|^2$.

(c) Suppose that $f[n]$ is produced by sampling $x(t)$ once every 0.1ms ($F_s = 10,000$ samples/second). Find $F(\omega)$, the DTFT of $f[n]$, and its associated power spectrum $|F(\omega)|^2$.

(d) Suppose that $g[n]$ is produced by sampling $y(t)$ once every 0.1ms, for a total of exactly ten pitch periods (thus there are a total of $N = 1000$ samples). Let $G[k]$ be the 1000-point DFT of $g[n]$. Find $G[k]$, and its associated power spectrum $|G[k]|^2$.

Problem 4.1.2

The vowel /i/ is characterized by formant frequencies at $F_1 = 300$ and $F_2 = 2000$ Hertz, and with bandwidths of roughly $B_1 = 150$ and $B_2 = 300$ Hertz. This problem will focus only on the positive-frequency part of the first formant ringing, and will ignore amplitude and phase, thus

$$x(t) = e^{-(\pi 150 - j2\pi 300)t}u(t)$$

(a) Find $X(\omega)$ and $|X(\omega)|^2$, the CTFT and its associated power spectrum.

(b) Suppose that $y(t)$ is the periodic repetition of $x(t)$, repeated once every 10ms. Find the Fourier series coefficients Y_k, and the associated power spectrum $|Y_k|^2$.

(c) Suppose that $f[n]$ is produced by sampling $x(t)$ once every 0.1ms ($F_s = 10,000$ samples/second). Find $F(\omega)$, the DTFT of $f[n]$, and its associated power spectrum $|F(\omega)|^2$.

(d) Suppose that $g[n]$ is produced by sampling $y(t)$ once every 0.1ms, for a total of exactly ten pitch periods (thus there are a total of $N = 1000$ samples). Let $G[k]$ be the 1000-point DFT of $g[n]$. Find $G[k]$, and its associated power spectrum $|G[k]|^2$.

Problem 4.1.3

The vowel /ɛ/ is characterized by formant frequencies at $F_1 = 600$ and $F_2 = 1700$ Hertz, and with bandwidths of roughly $B_1 = 150$ and $B_2 = 250$ Hertz. This problem will focus only on the positive-frequency part of the first formant ringing, and will ignore amplitude and phase, thus

$$x(t) = e^{-(\pi 150 - j2\pi 600)t}u(t)$$

(a) Find $X(\omega)$ and $|X(\omega)|^2$, the CTFT and its associated power spectrum.

(b) Suppose that $y(t)$ is the periodic repetition of $x(t)$, repeated once every 10ms. Find the Fourier series coefficients Y_k, and the associated power spectrum $|Y_k|^2$.

(c) Suppose that $f[n]$ is produced by sampling $x(t)$ once every 0.1ms ($F_s = 10,000$ samples/second). Find $F(\omega)$, the DTFT of $f[n]$, and its associated power spectrum $|F(\omega)|^2$.

(d) Suppose that $g[n]$ is produced by sampling $y(t)$ once every 0.1ms, for a total of exactly ten pitch periods (thus there are a total of $N = 1000$ samples). Let $G[k]$ be the 1000-point DFT of $g[n]$. Find $G[k]$, and its associated power spectrum $|G[k]|^2$.