UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 498MH Signal and Image Analysis

Homework 3

Fall 2014

Assigned: Thursday, September 11, 2014 — Due: Thursday, September 25, 2014

Reading: Mark Hasegawa-Johnson, *Lecture Notes in Speech Production, Speech Coding and Speech Recognition*, Chapter 1: Basics of Digital Signal Processing,
`http://isle.illinois.edu/~hasegawa/notes/chap1.pdf`

1 Fourier Transform

Do **one** of the following three problems.

Problem 3.1.1

The vowel /a/ is characterized by formant frequencies at $F_1 = 900$ and $F_2 = 1100$ Hertz, and with bandwidths of roughly $B_1 = B_2 = 150$ Hertz. This means that, every time the glottis closes, it excites a ringing in the vocal tract that sounds like

$$x(t) = \begin{cases} 0 & t < 0 \\ A_1 e^{-2\pi 75t} \sin(2\pi 900t) + A_2 e^{-2\pi 75t} \sin(2\pi 1100t) & t \geq 0 \end{cases}$$

for some amplitudes A_1 and A_2.

(a) Find the Fourier transform of $x(t)$. You can express your answer as either $X(\omega)$, where ω is in radians/second (as we have been doing in class), or as $X(f)$, where f is in Hertz (as is done in the recommended reading for this problem set). Your answer should be the sum of four terms.

(b) Calculate and draw the power spectrum corresponding to just one of the terms, specifically if

$$y(t) = \frac{A_1}{2j} e^{-2\pi(75+900j)t}, \quad t \geq 0$$

then calculate and draw its power spectrum, $|Y(\omega)|^2$. Your drawing should show the value of the power spectrum, $|Y(\omega)|^2$ in terms of A_1, at the frequencies $\omega = 0$, $\omega = 2\pi 900$, and $\omega \to \infty$.

Problem 3.1.2

The vowel /i/ is characterized by formant frequencies at $F_1 = 300$ and $F_2 = 2000$ Hertz, and with bandwidths of roughly $B_1 = 150$ and $B_2 = 300$ Hertz. This means that, every time the glottis closes, it excites a ringing in the vocal tract that sounds like

$$x(t) = \begin{cases} 0 & t < 0 \\ A_1 e^{-2\pi 75t} \sin(2\pi 300t) + A_2 e^{-2\pi 150t} \sin(2\pi 2000t) & t \geq 0 \end{cases}$$

for some amplitudes A_1 and A_2.

(a) Find the Fourier transform of $x(t)$. You can express your answer as either $X(\omega)$, where ω is in radians/second (as we have been doing in class), or as $X(f)$, where f is in Hertz (as is done in the recommended reading for this problem set). Your answer should be the sum of four terms.

(b) Calculate and draw the power spectrum corresponding to just one of the terms, specifically if

$$y(t) = \frac{A_1}{2j} e^{-2\pi(75+300j)t}, \quad t \geq 0$$

then calculate and draw its power spectrum, $|Y(\omega)|^2$. Your drawing should show the value of the power spectrum, $|Y(\omega)|^2$ in terms of A_1, at the frequencies $\omega = 0$, $\omega = 2\pi 300$, and $\omega \to \infty$.

Problem 3.1.3

The vowel /ɛ/ is characterized by formant frequencies at $F_1 = 600$ and $F_2 = 1700$ Hertz, and with bandwidths of roughly $B_1 = 150$ and $B_2 = 250$ Hertz. This means that, every time the glottis closes, it excites a ringing in the vocal tract that sounds like

$$x(t) = \begin{cases} 0 & t < 0 \\ A_1 e^{-2\pi 75t} \sin(2\pi 600t) + A_2 e^{-2\pi 125t} \sin(2\pi 1700t) & t \geq 0 \end{cases}$$

for some amplitudes A_1 and A_2.

(a) Find the Fourier transform of $x(t)$. You can express your answer as either $X(\omega)$, where ω is in radians/second (as we have been doing in class), or as $X(f)$, where f is in Hertz (as is done in the recommended reading for this problem set). Your answer should be the sum of four terms.

(b) Calculate and draw the power spectrum corresponding to just one of the terms, specifically if

$$y(t) = \frac{A_1}{2j} e^{-2\pi(75+600j)t}, \quad t \geq 0$$

then calculate and draw its power spectrum, $|Y(\omega)|^2$. Your drawing should show the value of the power spectrum, $|Y(\omega)|^2$ in terms of A_1, at the frequencies $\omega = 0$, $\omega = 2\pi 500$, and $\omega \to \infty$.

2 Many Transforms

Do **one** of the following three problems.

Problem 3.2.1

Using the methods we've learned so far, each of the following signals has either a CTFS, CTFT, DTFS/DFT, or DTFT. No signal has more than one of these things (next week we will learn a special notation called an "impulse" that allows you to represent one of these in terms of the others). For each of these four signals, compute its CTFS, CTFT, DFT, or DTFT, whichever one is possible. Remember that there's only one integral you need to know:

$$\int e^{\alpha t} dt = \frac{1}{\alpha} e^{\alpha t} \text{ if } |\alpha| < 1$$

and there's only one infinite summation you need to know:

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \quad \text{if } |a| < 1$$

Modification 9/8/2014: You will only be graded on the two continuous-time signals (functions of t), not on the discrete-time signals (functions of n). Give the discrete-time signals a try if you wish; your attempt will not be graded.

(a)

$$x_a[n] = \begin{cases} \left(\frac{3}{4}\right)^n & 0 \le n \le 45 \\ x[n-46] & \text{always} \end{cases}$$

(b)

$$x_b(t) = \begin{cases} e^{-2\pi 200t} \cos(2\pi 1000t) & 0 \le t < 0.1 \\ x(t-0.1) & \text{always} \end{cases}$$

(c)

$$x_c(t) = \begin{cases} e^{-2\pi 200t} \cos(2\pi 1000t) & 0 \le t \\ 0 & \text{otherwise} \end{cases}$$

(d)

$$x_d[n] = \begin{cases} \left(\frac{3}{4}\right)^n & 0 \le n \\ 0 & \text{otherwise} \end{cases}$$

Problem 3.2.2

Using the methods we've learned so far, each of the following signals has either a CTFS, CTFT, DTFS/DFT, or DTFT. No signal has more than one of these things (next week we will learn a special notation called an "impulse" that allows you to represent one of these in terms of the others). For each of these four signals, compute its CTFS, CTFT, DFT, or DTFT, whichever one is possible. Remember that there's only one integral you need to know:

$$\int e^{\alpha t} dt = \frac{1}{\alpha} e^{\alpha t} \quad \text{if } |\alpha| < 1$$

and there's only one infinite summation you need to know:

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \quad \text{if } |a| < 1$$

Modification 9/8/2014: You will only be graded on the two continuous-time signals (functions of t), not on the discrete-time signals (functions of n). Give the discrete-time signals a try if you wish; your attempt will not be graded.

(a)

$$x_a[n] = \begin{cases} \left(\frac{1}{2}\right)^n & 0 \le n \\ 0 & \text{otherwise} \end{cases}$$

(b)

$$x_b[n] = \begin{cases} \left(\frac{1}{2}\right)^n & 0 \le n \le 19 \\ x[n-20] & \text{always} \end{cases}$$

(c)

$$x_c(t) = \begin{cases} e^{-2\pi 250t}\cos(2\pi 800t) & 0 \le t \\ 0 & \text{otherwise} \end{cases}$$

(d)

$$x_d(t) = \begin{cases} e^{-2\pi 250t}\cos(2\pi 800t) & 0 \le t < 0.2 \\ x(t-0.2) & \text{always} \end{cases}$$

Problem 3.2.3

Using the methods we've learned so far, each of the following signals has either a CTFS, CTFT, DTFS/DFT, or DTFT. No signal has more than one of these things (next week we will learn a special notation called an "impulse" that allows you to represent one of these in terms of the others). For each of these four signals, compute its CTFS, CTFT, DFT, or DTFT, whichever one is possible. Remember that there's only one integral you need to know:

$$\int e^{\alpha t} dt = \frac{1}{\alpha} e^{\alpha t} \quad \text{if } |\alpha| < 1$$

and there's only one infinite summation you need to know:

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \quad \text{if } |a| < 1$$

Modification 9/8/2014: You will only be graded on the two continuous-time signals (functions of t), not on the discrete-time signals (functions of n). Give the discrete-time signals a try if you wish; your attempt will not be graded.

(a)

$$x_a[n] = \begin{cases} \left(\frac{2}{3}\right)^n & 0 \le n \le 14 \\ x[n-15] & \text{always} \end{cases}$$

(b)

$$x_b[n] = \begin{cases} \left(\frac{2}{3}\right)^n & 0 \le n \\ 0 & \text{otherwise} \end{cases}$$

(c)

$$x_c(t) = \begin{cases} e^{-2\pi 50t}\cos(2\pi 500t) & 0 \le t < 0.05 \\ x(t-0.05) & \text{always} \end{cases}$$

(d)

$$x_d(t) = \begin{cases} e^{-2\pi 50t}\cos(2\pi 500t) & 0 \le t \\ 0 & \text{otherwise} \end{cases}$$