UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 498MH Principles of Signal Analysis
Fall 2014

MIDTERM EXAM 2 SOLUTIONS

Problem 1 (25 points)

Consider the system $y[n] = \begin{cases} x[n] & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$

(a) Is this system linear? Prove your answer.

SOLUTION: Yes, because

$$x_3[n] = ax_1[n] + bx_2[n] \rightarrow y_3[n] = \begin{cases} ax_{[}n] + bx_2[n] & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

$$ay_1[n] + by_2[n] = \begin{cases} ax_1[n] + bx_2[n] & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

(b) Is this system time-invariant? Prove your answer.

SOLUTION: No, because

$$x_3[n] = x_1[n-m] \rightarrow y_3[n] = \begin{cases} x_1[n-m] & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

$$y_1[n-m] = \begin{cases} x_1[n-m] & n-m \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

Problem 2 (25 points)

A particular LTI system has the impulse response

$$h[n] = \delta[n] + \delta[n-2]$$

(a) What is the frequency response, $H(\omega)$, of this system?

SOLUTION: $H(\omega) = 1 + e^{-2j\omega} = e^{-j\omega} \cos\omega$

(b) Suppose the input is $x[n] = \delta[n] + \delta[n-3]$. What is the output?

SOLUTION: $y[n] = \begin{cases} 1 & n \in \{0, 2, 3, 5\} \\ 0 & \text{otherwise} \end{cases}$

Problem 3 (25 points)

Suppose you have a signal sampled at $F_s = 600$ samples/second. You wish to create a notch filter to eliminate a noise component at 60Hz. You choose to do this using the following filter:

$$H(z) = \frac{(1 - r_1 z^{-1})(1 - r_2 z^{-1})}{(1 - p_1 z^{-1})(1 - p_2 z^{-1})}$$

(a) Specify the values of the poles p_1, p_2 and the zeros r_1, r_2. Note that there is one free parameter in your answer that is not specified by the problem statement; you may set that free parameter to any reasonable value.

SOLUTION: $r_1 = e^{j\pi/5}$, $r_2 = e^{-j\pi/5}$, $p_1 = 0.99e^{j\pi/5}$, $p_2 = 0.99e^{-j\pi/5}$

(b) Now suppose you are given a system function

$$H(z) = \frac{(1 - r_1 z^{-1})(1 - r_2 z^{-1})}{(1 - p_1 z^{-1})(1 - p_2 z^{-1})}$$

and you wish to implement this using the equation

$$y[n] = x[n] + b_1 x[n-1] + b_2 x[n-2] - a_1 y[n-1] - a_2 y[n-2]$$

Find b_1, b_2, a_1 and a_2 in terms of r_1, r_2, p_1 and p_2.

SOLUTION: $b_1 = -(r_1 + r_2)$, $b_2 = r_1 r_2$, $a_1 = -(p_1 + p_2)$, $a_2 = p_1 p_2$

Problem 4 (25 points)

Suppose you want to design a band-stop filter as follows: $D(\omega) = \begin{cases} 1 & |\omega| < \frac{\pi}{5} \\ 0 & \frac{\pi}{5} < |\omega| < \frac{\pi}{2} \\ 1 & \text{otherwise} \end{cases}$

(a) Find the desired impulse response, $d[n]$.

SOLUTION: $d[n] = \text{sinc}(\pi n) - \left(\frac{1}{2}\right)\text{sinc}\left(\frac{\pi n}{2}\right) + \left(\frac{1}{5}\right)\text{sinc}\left(\frac{\pi n}{5}\right)$

(b) Suppose you are willing to tolerate transition bands of up to $\Delta\omega = \frac{\pi}{32}$ radians/sample (measured from passband ripple to stopband ripple), but that you want the ripples to be as small as possible. What type of window should you use, and how long should it be?

SOLUTION: Hamming window, with a length of $N = 256$ samples.