

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN  
Department of Electrical and Computer Engineering  
ECE 498MH SIGNAL AND IMAGE ANALYSIS

**Solution 2**  
Fall 2013

Assigned: Friday, September 13, 2013

Due: Friday, September 20, 2013

Reading: Signal Processing First (SPF) pp. 36-57

**Problem 2.1**

Consider the following signal:

$$x[n] = 2 + 2 \cos \frac{\pi n}{4} + \sin \frac{\pi n}{2} + \frac{1}{2} \cos \frac{3\pi n}{4}$$

(a)

$$X_k = \begin{cases} 2 & k = 0 \\ 1 & k = \pm 1 \\ \frac{1}{2j} & k = 2 \\ -\frac{1}{2j} & k = -2 \\ \frac{1}{4} & k = \pm 3 \end{cases}$$

(b) Sketch should show

$$|X_k|^2 = \begin{cases} 4 & k = 0 \\ 1 & k = \pm 1 \\ \frac{1}{4} & k = \pm 2 \\ \frac{1}{16} & k = \pm 3 \end{cases}$$

(c) Total power is  $4 + 1 + 1 + \frac{1}{4} + \frac{1}{4} + \frac{1}{16} + \frac{1}{16} = 6\frac{5}{8}$ .

**Problem 2.2**

Consider the signal

$$x(t) = |\cos(2\pi t)|$$

(a) Sketch is like a cosine, but all positive.

(b)  $T_0 = \frac{1}{2}$ ,  $\Omega_0 = 4\pi$ .

(c)

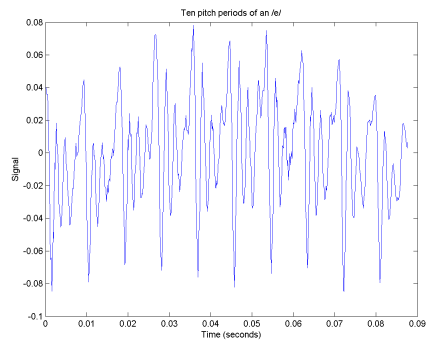
$$X_k = \begin{cases} \frac{2}{\pi} & k = 0 \\ \frac{2}{\pi} \int_{-1/4}^{1/4} \frac{e^{j2\pi t} + e^{-j2\pi t}}{2} e^{-j\pi k t} dt & k \neq 0 \end{cases}$$

$$X_k = \begin{cases} \frac{2}{\pi} & k = 0 \\ \frac{1}{(1-2k)\pi} \sin\left(\frac{1-2k}{2}\pi\right) + \frac{1}{(1+2k)\pi} \sin\left(\frac{1+2k}{2}\pi\right) & k \neq 0 \end{cases}$$

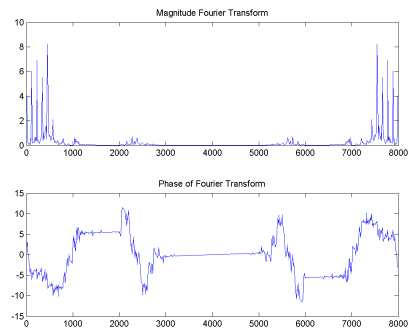
## Matlab Exercises

## Problem 2.3

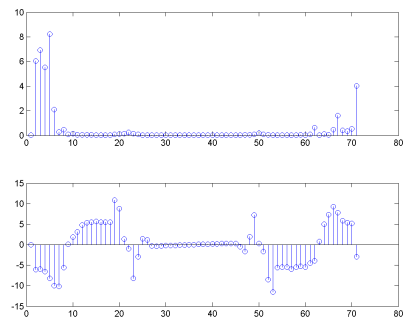
(a)



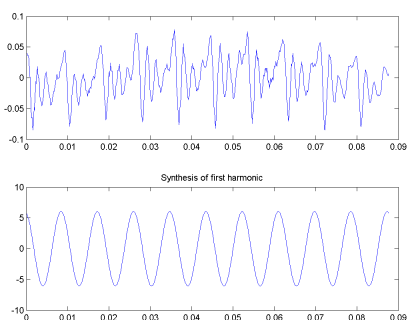
(b)

(c)  $T_0 = 0.0088$ seconds for this waveform.

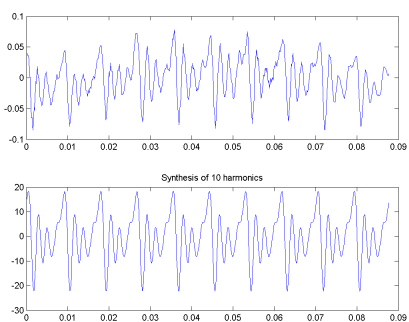
(d)



(e)



(f)



(g)

- (h) Pitch is retained by the one-cosine approximation. Vowel quality starts to be audible with the two-cosine or five-cosine approximation.