UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 498MH Signal and Image Analysis

Solutions 6

Fall 2013

Assigned: Friday, October 18, 2013 Due: Friday, November 1, 2013

Reading: SPF Chapters 7 and 8

Problem 6.1

This problem will consider the DTFT of several windows, where the DTFT transform pair is defined as

$$\begin{aligned} X(\omega) &= \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \\ x[n] &= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)e^{j\omega n}d\omega \end{aligned}$$

(a) Use Zeno's paradox ($\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$) to compute the DTFT of

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

Notice that your answer is only valid for $\omega \neq 0$, because the DTFT goes to infinity at $\omega = 0$.

(b) Use your answer to part (a), together with the time-shift property of the Z-transform, to find the DTFT $R(\omega)$ of the rectangular window $r[n]$, where

$$r[n] = u[n+M] - u[n-(M+1)] = \begin{cases} 1 & -M \leq n \leq M \\ 0 & \text{otherwise} \end{cases}$$

Notice that $r[n]$ is symmetric in time, so your answer should be real-valued!

(c) What is the first null of $R(\omega)$ (the first frequency at which $R(\omega) = 0$)?

(d) The first side-lobe of $R(\omega)$ is at the frequency $\omega = \frac{3\pi}{2M+1}$. What is its value, $R\left(\frac{3\pi}{2M+1}\right)$?

Matlab Exercises

Problem 6.2

Download the data file signal.dat from `http://www.ifp.illinois.edu/~jones/ece598/signal.dat`. (You can click on the hyperlink, then Save the page source.) Load it:
`load signal.dat;`
the data will be loaded into a vector called signal.

Plot the magnitude and phase of the DFT (or FFT) of this signal on separate stem or sample plots. For the rest of this assignment, plot all magnitudes twice, once on a linear scale (`abs(signalDFT)`) and on another plot in decibels (`20*log10(abs(signalDFT))`).

Plot the magnitude and phase corresponding to the non-negative frequencies of this signal zero-padded to 1024 samples. (Since zero-padding interpolates the continuous spectrum, this time use a normal line plot.) Scale the horizontal axis to display the proper DTFT frequencies. (Hint: create a vector `freqvals` of the same number of samples as the zero-padded DFT points you intend to display, and use `plot(freqvals,dftmags)`.)

The signal consists of five sinusoids of various frequencies and amplitudes. Using various (length-64!) windows and zero-padding to visualize the magnitude of the spectrum, determine the frequencies and magnitudes of all five of these sinusoids to the best of your ability. The online module `http://cnx.org/content/m12032/latest/` about spectral analysis using windows and the DFT may be a helpful guide for this exercise. The Matlab scripts windows.m and zeropad.m, which generated many of the figures in that module, can be modified to solve this exercise:

`http://www.ifp.illinois.edu/~jones/ece598/windows.m`

`http://www.ifp.illinois.edu/~jones/ece598/zeropad.m`

Turn in the plots as your answer, along with a list of the frequency and magnitude values you determined (and the plots supporting those estimates).