UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 498MH Signal and Image Analysis

Homework 1

Fall 2013

Assigned: Friday, September 6, 2013 Due: Friday, September 13, 2013

Reading: McClellan & Schafer 2.1-2.5

Problem 1.1

Find the real part of each of the following functions.

(a) $236\exp\left(j\left(\frac{\pi n}{2}+\frac{\pi}{2}\right)\right)$

(b) $-j\exp\left(j\pi n/16\right)$ (take advantage of the fact that $j=e^{j\pi/2}$)

Problem 1.2

Write each of the following functions as $\Re\left\{Ae^{j\omega n}e^{j\phi}\right\}$ for some A, ω and ϕ:

(a) $544\cos\left(\frac{\pi n}{3}\right)$

(b) $-26\sin\left(\frac{\pi n}{10}-\frac{\pi}{4}\right)$

(c) $\cos\left(\frac{\pi n}{10}\right)+3\sqrt{2}\sin\left(\frac{\pi n}{10}+\frac{\pi}{4}\right)$

Problem 1.3

Each of the following functions may be written as

$$C\cos(\omega n+\psi)=\Re\left\{Ce^{j\psi}e^{j\omega n}\right\}=\Re\left\{(Ae^{j\theta}+Be^{j\phi})e^{j\omega n}\right\}$$

for some $A, B, C, \theta, \phi, \psi$ and ω. For each of these functions, **draw a phasor diagram** (imaginary part versus real part) showing the phasor addition $Ce^{j\psi}=Ae^{j\theta}+Be^{j\phi}$. You will be graded on having each phasor having roughly the right magnitude and angle; you will not be graded on the exact final value.

(a) $\cos\left(\frac{\pi n}{2}\right)+\cos\left(\frac{\pi n}{2}-\frac{\pi}{2}\right)$

(b) $\cos\left(\frac{\pi n}{2}\right)+\sin\left(\frac{\pi n}{2}\right)$

(c) $\cos\left(\frac{\pi n}{10}\right)+3\sqrt{2}\sin\left(\frac{\pi n}{10}+\frac{\pi}{4}\right)$

Problem 1.4

The following is the equation for a 3000Hz pure tone, as a function of t in seconds:

$$x(t)=\cos\left(2\pi 3000t\right)$$

Suppose that this signal is sampled at one sampling frequency F_1 samples/second, in order to construct the following digital signal:

$$y[n] = x\left(\frac{n}{F_1}\right)$$

Unfortunately, the guy doing the recording forgot to keep track of the sampling frequency, so it gets played back at *the wrong sampling frequency*, resulting in a signal $z(t)$ that has these sample values:

$$z\left(\frac{n}{F_2}\right) = y[n]$$

For each of the following input and output sampling frequencies, find the signal $z(t)$ that gets played back. In one of these two sub-parts, the only mistake will be $F_1 \neq F_2$; in the other sub-part, there will be a further mistake caused by aliasing because F_1 doesn't satisfy the Nyquist criterion.

(a) $F_1 = 4000$Hz, $F_2 = 6000$Hz

(b) $F_1 = 8000$Hz, $F_2 = 12000$Hz

Matlab Exercises

Problem 1.5

For each of the four sub-parts of this problem, create a figure with six sub-plots (thus you have a total of 24 sub-plots). For each sub-plot, set the parameters `subplotnumber`, `omega`, `phi`, and `A` as specified by the problem, then run the following commands:

```
n=[0:9];
x=A*cos(omega*n+phi);
subplot(3,2,subplotnumber);
stem(n,x);
title(sprintf('x[n] with omega=\%d, phi=\%d, A=\%d',omega,phi,A));
```

Turn in all five of your figures. The best way to do this (least wasteful of paper) is to first print each figure as a GIF (`print -dGIF figureN.gif;`, where `N` is the figure number), then include the GIF images in a Word document, scale the images so that two or three of them fit on each page (but make sure the axis labels are still clear!!!!), and then print the Word document.

(a) Use `A=1;phi=0;` and `omega` set to each of the following six values: $\left\{0, \frac{2\pi}{10}, \frac{4\pi}{10}, \pi, \frac{16\pi}{10}, 2\pi\right\}$.

(b) Use `A=1;omega=2*pi/10;` and set phi equal to each of the following five values: $\left\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi\right\}$. For the sixth and last sub-plot, set phi equal to zero, but use sin instead of cos.

(c) In three sub-plots, plot `x=A*cos(omega*n+phi)` for `omega=2*pi/10;phi=0` and for $A \in \{0.1, 1, 10\}$. For the other three sub-plots, plot

```
xsquared=(A*cos(omega*n+phi)).^2;
```

for each of the same three settings.

(d) Plot `x=A*cos(omega*(n+d));` for `A=1;omega=2*pi/10;` and for values of $d \in \{0, 1, 2, 3, 4, 5\}$.

Problem 1.6

For each of the three sub-parts in written problem 1.3, create a phasor plot showing the phasor addition, and hand in the plot. If you set the variables `A, B, phi, and theta`, you can show the addition using the following code:

```
z = [0,A*exp(j*theta),A*exp(j*theta)+B*exp(j*phi)];
plot(real(z),imag(z),'-x');
```

Problem 1.7

For each of the sub-parts of this problem, set the variables `A`, `f`, and `phi` as specified, then use the following code to play a quarter-second waveform at $F_s = 8000$ samples/second, and then answer the question in the problem statement.

```
FS=8000;
n=[0:(FS/4)];
x=A*cos(phi+2*pi*n*f/FS);
sound(x,FS);
```

(a) Use `A=1;phi=0;` and values of `f=500`, `f=1000`, and `f=2000`. Answer the question: changing the frequency of a tone changes its perceived pitch, its perceived loudness, or nothing at all?

(b) Use `f=1000;phi=0;` and values of `A=0.01`, `A=0.1`, and `A=1`. Answer the question: changing the amplitude of a tone changes its perceived pitch, its perceived loudness, or nothing at all?

(c) Use `f=1000;A=1;` and values of `phi=0`, `phi=pi/4`, and `phi=pi/2`. Answer the question: changing the phase of a tone changes its perceived pitch, its perceived loudness, or nothing at all?

Problem 1.8

In this problem, you will create a variety of 250×250 images using the `repmat` function in matlab. Each of these images will be based on the vector `x=128+127*cos(2*pi*[0:249]/249);`

(a) Create the image `A=repmat(x,[250,1]);` Use `size(A)` to verify that `A` is a 250×250 matrix. Use `A(1:6,1:6);` to print out the first six rows and first six columns of `A`; have you created an image with horizontal stripes, or vertical stripes? Use `imagesc(A);` to show the image and verify your conclusion. Use `print -dgif imageA.gif` to print it, and include it in your lab report.

(b) Create the image `B=repmat(x',[1,250]);` Use `size(B)` to verify that `B` is a 250×250 matrix. Use `B(1:6,1:6);` to print out the first six rows and first six columns of `B`; have you created an image with horizontal stripes, or vertical stripes? Use `imagesc(B);` to show the image, and verify your conclusion. Use `print -dgif imageB.gif` to print it, and include it in your lab report.

(c) Use `for n1=0:249, for n2=0:249, C(n1,n2)=128+127*cos(2*pi*(n1+n2)/249); end; end` to create an image with diagonal stripes. Use `size(C)` to verify that `C` is a 250×250 matrix. Use `C(1:6,1:6)` to print out the first six rows and first six columns of `C`, and observe how the stripes are now diagonal. Use `imagesc(C);` to show the image. Use `print -dgif imageC.gif` to print it, and include it in your lab report.

(d) Use `D=cat(3,A,B,C);` to create an image with three layers: a red layer specified by image A, a green layer specified by image B, and a blue layer specified by image C. Use `size(D)` to verify that the result is a $250 \times 250 \times 3$ array. Use `D(1:6,1:6,1:3)` to show the first six rows and first six columns in each of the three layers; verify that these are the same things you saw in parts (a)-(c). Now use `imagesc(D);` to show the image. Use `print -dgif imageD.gif` to print it, and include it in your lab report.