UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 498MH Principles of Signal Analysis
Fall 2013

MIDTERM EXAM 2

Wednesday, November 6, 2013

- This is a CLOSED BOOK exam.

- There are a total of 100 points in the exam. Each problem specifies its point total. Plan your work accordingly.

- You must SHOW YOUR WORK to get full credit.

Problem	Score
1	
2	
3	
4	
5	
Total	

Name: _____

θ	$\cos\theta$	$\sin\theta$	$e^{j\theta}$
0	1	0	1
$\pi/6$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}/2 + j/2$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2 + j\sqrt{2}/2$
$\pi/3$	$1/2$	$\sqrt{3}/2$	$1/2 + j\sqrt{3}/2$
$\pi/2$	0	1	j
π	-1	0	-1
$3\pi/2$	1	-1	$-j$
2π	1	0	1

Problem 1 (20 points)

A signal is corrupted with narrowband noise at $\omega_n = \frac{\pi}{6}$ radians/sample. To remove the noise, you create a notch filter with a pole magnitude of $r = 0.9$. The filter system function is

$$H(z) = \frac{(1 - r_1 z^{-1})(1 - r_2 z^{-1})}{(1 - p_1 z^{-1})(1 - p_2 z^{-1})}$$

Speciy the poles and zeros in magnitude/phase form (e.g., $p_1 = m e^{j\theta}$, but you should specify the numerical value of m and the numerical value of θ).

$$r_1 \quad = \quad \boxed{}$$

$$r_2 \quad = \quad \boxed{}$$

$$p_1 \quad = \quad \boxed{}$$

$$p_2 \quad = \quad \boxed{}$$

Problem 2   (20 points)

Suppose you want to implement a filter with the following frequency response:

$$H(\omega) = \frac{(1 - 0.5e^{j\pi/4}e^{-j\omega})(1 - 0.5e^{-j\pi/4}e^{-j\omega})}{(1 + 0.5e^{j\pi/4}e^{-j\omega})(1 + 0.5e^{-j\pi/4}e^{-j\omega})}$$

You realize that you can get this frequency response by writing one line of code in matlab, implementing the following equation:

$$y[n] = x[n] + b_1 x[n - 1] + b_2 x[n - 2] + a_1 y[n - 1] + a_2 y[n - 2]$$

Find the filter coefficients:

$a_1 \quad = \quad \boxed{}$

$a_2 \quad = \quad \boxed{}$

$b_1 \quad = \quad \boxed{}$

$b_2 \quad = \quad \boxed{}$

Problem 3   (20 points)

An FIR filter will be created as $h[n] = w[n]d[n]$, for $D(\omega) = \begin{cases} 1 & |\omega| < \frac{\pi}{3} \\ 0 & \text{else} \end{cases}$. You are considering three possible windows: rectangular, Hamming, and Hann, each of length $2M+1 = 23$ samples. These three windows are differentiated by three typical measurements, depicted in the figure below: transition width, band-edge ripple, and distant ripple.
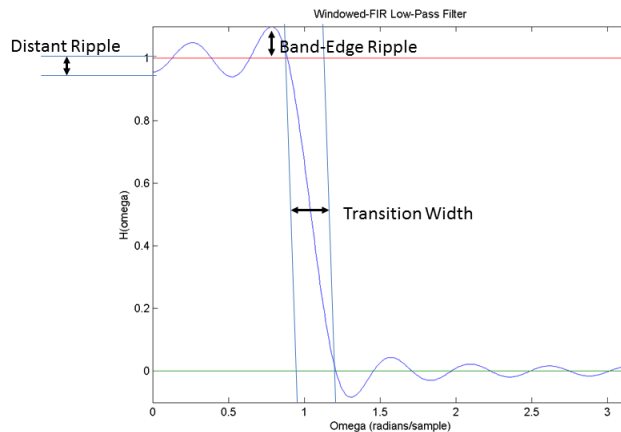
Your evil cousin Morwen has calculated transition width, band-edge ripple, and distant ripple for the rectangular, Hamming, and Hann windows. Out of spite, however, she has mixed them up, so that you don't know which one goes with which window; all you know is that, for each of the three windows,

$$
\begin{array}{rl}
\text{Transition Width} & \text{is either} \quad \dfrac{2\pi}{23} \text{ or } \dfrac{4\pi}{23} \\
\text{Band-Edge Ripple} & \text{is either} \quad 0.0033 \text{ or } 0.0062 \text{ or } 0.099 \\
\text{Distant Ripple} & \text{is either} \quad 0.0006 \text{ or } 0.0031 \text{ or } 0.045
\end{array}
$$

Sort the values above into the table below.

Window	Rectangular	Hamming	Hann
Transition Width			
Band-Edge Ripple			
Distant Ripple			

Problem 4   (20 points)

Consider

$$x[n] \rightarrow \boxed{\text{System}} \rightarrow y[n]$$

where the system is defined by the relationship $y[n] = w[n]x[n]$ for

$$w[n] = \begin{cases} 1 & |n| \le 7 \\ 0 & \text{otherwise} \end{cases}$$

Prove that this system is **not** time-invariant.

Problem 5   (20 points)

Consider the signal $y[n]$ defined by

$$y[n] = \begin{cases} e^{j0.42\pi n} & |n| \leq 11 \\ 0 & \text{otherwise} \end{cases}$$

Use the windowing property $(w[n]x[n] \leftrightarrow \frac{1}{2\pi}W(\omega) * X(\omega))$ to find $Y(\omega)$, the DTFT of $y[n]$.