UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 410 Digital Signal Processing

# Quiz 4

Professors Bresler, Hasegawa-Johnson — Wednesday October 20th, 2010

**Name:** — **Section: 1pm (E) or 3pm(G)**

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## Instructions

- You may not use any calculators, cell phones, earphones, or any other forms of electronics on this quiz.
- Show all your work to receive full credit for your answers.
- When you are asked to "calculate", "determine", or "find", this means providing closed-form expressions (i.e., without summation or integration signs).
- Neatness counts. If we are unable to read your work, we cannot grade it.
- Turn in your entire booklet once you are finished. No extra booklet or papers will be considered.
- The asterisk mark ($*$) denotes convolution.

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| Problem | Points |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| Total | |

**Problem 1** (28 points)

(a) (9 points) For each of the systems described below, find a possible expression for the transfer function. Explain why it is not the only one possible, and how alternative solutions are related to the one you provided.

$(H_1)$ : zeros: $-1$, $1$; poles: $1/4$, $2$

$(H_2)$ : zeros: $1/4$; $2$; poles: $\sqrt{2}j/2$, $-\sqrt{2}j/2$, $-1$

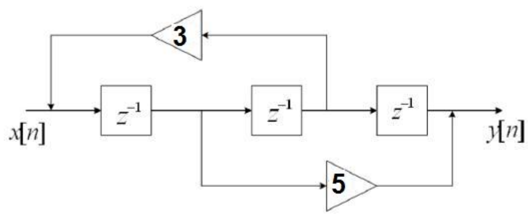
$(H_3)$ : zeros: $1/2$, $1/3$; poles: $2$, $-2$

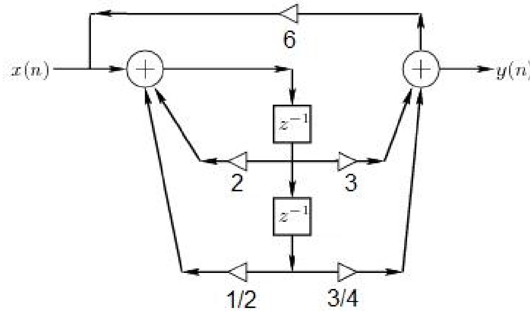
(b) (9 points) For each of the systems, determine whether it is BIBO stable.

(c) (10 points) Select two of the three systems from part (a) so that when they are connected in cascade, the resulting system is BIBO stable. Find its transfer function and plot its pole-zero graph.

**Problem 2** (25 points)

Consider the following causal systems.




(a) (10 points) Find the transfer functions of each system. Call the system on the left $H_1$ and the system on the right $H_2$.

(b) (7 points) Find the difference equation of each system.

(c) (8 points) Determine whether each system is BIBO Stable. For each case that the system is determined to be unstable, find a real-valued *bounded* input that will produce an unbounded input. If you were unable to find $H_1$ and $H_2$ in part (a), please answer the question using the following systems:

$$G_1(z) = \frac{3z}{z^3 + j}$$

$$G_2(z) = \frac{z-1}{z^2 + 3z + 2}$$

**Problem 3** (25 points)
Consider the following system with sampling period $T$ and an ideal D/A converter:

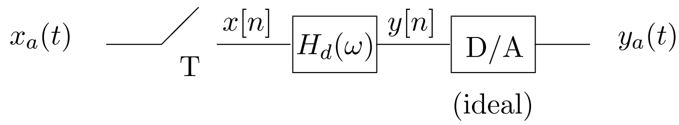



Figure 1: System for discrete-time processing of continuous time signals

The audio signal, $x_a(t)$, is assumed to be bandlimited to 25 kHz. It is desired to filter the signal with a bandstop filter that will suppress frequencies roughly about the musical note B-flat 3 (450 to 490 Hz) by using a digital filter, $H_d(\omega)$, as shown in the figure above.

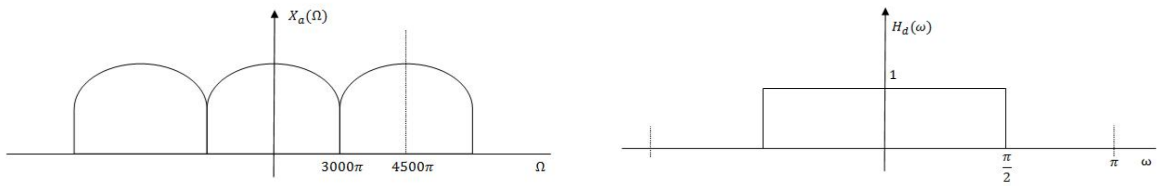
(a) (5 points) Determine the sampling period, $T_s$, to avoid aliasing.

(b) (10 points) Sketch the frequency response of the filter, $H_d(\omega)$, for the necessary discrete-time filter, when sampling at the Nyquist rate.

(c) (10 points) Repeat part (b), with the new sampling rate being 20 kHz and the chosen suppressed musical note now C-sharp 4 (250 to 280 Hz).

**Problem 4** (22 points)

The signal $x_a(t)$ is passed through a system as shown in the figure in problem 3. Shown below is the CTFT of $x_a(t)$, which is to be sampled at $T = 1/3$ msec.




(a) (12 points) Sketch $X_d(\omega)$, $Y_d(\omega)$, $Y_a(\Omega)$, assuming ideal D/A converter.

(b) (10 points) Suppose now that the ideal D/A converter is replaced with a first-order hold such that:

$$y_a(t) = \sum_{n=-\infty}^{\infty} y[n] p_1(t - nT)$$

where

$$p_1(t) = \begin{cases} \frac{1}{T}\left(1 - \frac{|t|}{T}\right) & |t| < T \\ 0 & |t| \geq T \end{cases}$$

For this system, find the relationship between $Y_a(\Omega)$ and $Y_d(\omega)$ (Hint: the pulse shaping function of the first-order hold is a scaled triangle function).