

ECE 498MH  
12/6/2013

## TOPICS

### ① PHASORS

PROBLEM  $A \cos(\omega t + \theta) + B \sin(\omega t + \phi) = C \cos(\omega t)$

WHAT ARE  $C$  AND  $\psi$ ?

SOLUTION  $A \cos(\omega t + \theta) = \operatorname{Re}(A e^{j\theta} e^{j\omega t})$

$$B \sin(\omega t + \phi) = \operatorname{Re}(-j B e^{j\phi} e^{j\omega t})$$

$$C \cos(\omega t + \psi) = \operatorname{Re}(A e^{j\theta} - j B e^{j\phi}) e^{j\omega t}$$

$$\Rightarrow C = |A e^{j\theta} - j B e^{j\phi}|$$

$$\psi = \angle(A e^{j\theta} - j B e^{j\phi})$$

### ② FOURIER SERIES

#### THEOREM

IF  $x(t)$  PERIODIC W/  $T_0$ , THEN  $x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j 2\pi k t / T_0}$

WITH POWER SPECTRUM  $|X_k|^2$

PROBLEM WHAT IS  $X_k$ ?

SOLUTION

CTFS

$$X_k = \frac{1}{T_0} \int x(t) e^{-j 2\pi k t / T_0} dt$$

DTFS

$$X_k = \frac{1}{N_0} \sum x[n] e^{-j 2\pi k n / N_0}$$

### 3 LTI SYSTEMS

THEOREM  $x_1[n] \rightarrow$  [SYSTEM]  $\rightarrow y_1[n]$

IF SYSTEM IS LTI,  $y_1[n] = \sum_{m=-\infty}^{\infty} h[m]x_1[n-m]$

PROBLEM HOW DO WE KNOW IF IT'S LTI?

SOLUTION

LINEAR

$$x_1[n] \rightarrow y_1[n]$$

$$x_2[n] \rightarrow y_2[n]$$

$$x_3[n] = ax_1[n] + bx_2[n] \rightarrow y_3[n]$$

[LINEAR IFF ALWAYS]

$$y_3[n] = ay_1[n] + by_2[n]$$

TI

$$x_1[n] \rightarrow y_1[n]$$

$$x_2[n] = x_1[n-d] \rightarrow y_2[n]$$

[TI IFF ALWAYS]

$$y_2[n] = y_1[n-d]$$

### 4 IMPULSE RESPONSE, FREQ RESPONSE

THEOREM

$$x[n] \rightarrow$$
 [LTI]  $\rightarrow y[n] = h[n] * x[n]$

$$e^{j\omega n} \rightarrow$$
 [LTI]  $\rightarrow H(\omega) e^{j\omega n}$

$$A \cos(\omega n + \theta) \rightarrow$$
 [LTI]  $\rightarrow A \cdot |H(\omega)| \cos(\omega n + \theta + \angle H(\omega))$

PROBLEM

WHAT IS  $h[n]$ ? WHAT IS  $H(\omega)$ ?

SOLUTION

EITHER

$$x[n] = \delta[n] \rightarrow \boxed{\text{LTI}} \rightarrow y[n] = h[n]$$

OR

$$x[n] = e^{j\omega n} \rightarrow \boxed{\text{LTI}} \rightarrow y[n]$$

THEN!

$$H(\omega) = \sum_{m=-\infty}^{\infty} h[m] e^{-j\omega m}$$

$$h[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(\omega) e^{j\omega n} d\omega$$

5 Z TRANSFORM

PROBLEM = How can you do  $e^{j\omega n} \rightarrow \boxed{\text{LTI}}$

FOR  $y[n] + a_1 y[n-1] + \dots + a_N y[n-N] = b_0 x[n] + \dots + b_M x[n-M]$

SOLUTION  $\mathcal{Z}\{a_1 f[n] + b_0 g[n]\} = a_1 F(z) + b_0 G(z)$

$$\mathcal{Z}\{y[n-1]\} = z^{-1} Y(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + \dots + b_M z^{-M}}{1 - a_1 z^{-1} - \dots - a_N z^{-N}}$$

$$H(\omega) = \frac{b_0 + b_1 e^{-j\omega} + \dots + b_M e^{-j\omega M}}{1 - a_1 e^{-j\omega} - \dots - a_N e^{-j\omega N}}$$

6 WINDOWING

PROBLEM  $y[n] = x[n] * h[n]$

$$z[n] = x[n] w[n]$$

SOLUTION  $Y(\omega) = X(\omega) H(\omega)$

$$Z(\omega) = \frac{1}{2\pi} X(\omega) * W(\omega)$$

7 UPSAMPLING & DOWNSAMPLING

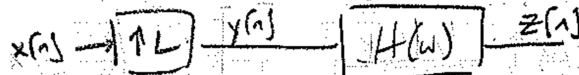
- PROBLEM
- CHANGE SAMPLING RATE
  - WHAT KIND OF ALIASING GETS INTRODUCED?

SOLUTION

(UP)

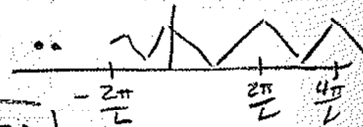
$$x[n] \rightarrow z[n] = \begin{cases} x[m], & n = mL \\ \text{INTERPOLATED,} & \text{OTHERWISE} \end{cases}$$

IS THE SAME AS

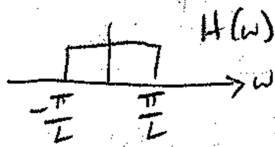


FOR WHICH

$$Y(\omega) = X(\omega L)$$



SO ALIASING IS ELIMINATED ONLY IF



$$h[n] = \text{sinc}\left(\frac{\pi n}{L}\right)$$

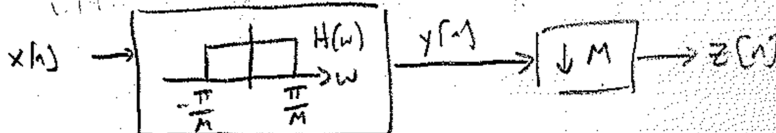
(DOWN)

$$y[n] \rightarrow \downarrow M \rightarrow z[n] = y[nM]$$

HAS THE TRANSFORM

$$Z(\omega) = \frac{1}{M} \sum_{k=0}^{M-1} Y\left(\frac{\omega + 2\pi k}{M}\right)$$

SO ALIASING AVOIDED IFF



8 SAMPLING

PROBLEM

$$y(t) \longrightarrow z[n] = y(nT)$$

OR

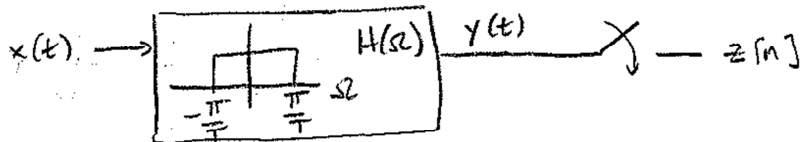
$$z[n] \longrightarrow x(t) = \sum_{n=-\infty}^{\infty} z[n] g(t-nT)$$

WHAT KIND OF ALIASING IS INTRODUCED?

SOLUTION

(A/D) 
$$Z(\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} Y\left(\frac{\omega + 2\pi k}{T}\right)$$

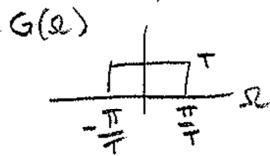
SO ALIASING AVOIDED ONLY IF



(D/A)

$$X(\Omega) = Z\left(\frac{\omega}{T}\right) G(\Omega)$$

SO ALIASING AVOIDED ONLY IF



$$\longleftrightarrow g(t) = \text{sinc}\left(\frac{\pi t}{T}\right)$$

9 LPC

SYNTHESIS MODEL

$$s[n] = e[n] + a_1 s[n-1] + \dots + a_p s[n-p]$$

ANALYSIS MODEL

$$e[n] = s[n] - a_1 s[n-1] - \dots - a_p s[n-p]$$

PROBLEM

$$\vec{a} = \begin{bmatrix} a_1 \\ \vdots \\ a_p \end{bmatrix} = \text{argmin} E[e^2[n]]$$

SOLUTION

$$\vec{a} = R^{-1} \vec{r}, \text{ E.G. IF } p=1, a_1 = \frac{R_{ss}[1]}{R_{ss}[0]}$$

## 10 WIENER FILTER

PROBLEM



NATURE

DESIGN  $h[n]$  TO MINIMIZE  $E[|z[n] - s[n]|^2]$

SOLUTION

$$H(\omega) = \frac{P_{sy}(\omega)}{P_{yy}(\omega)} = \frac{P_{ss}(\omega)}{P_{ss}(\omega) + P_{vv}(\omega)}$$

IF  $s[n]$  &  $v[n]$   
ARE UNCORRELATED  
AND ZERO-MEAN  
SIGNALS

OTHER FILTERS TO KNOW

- NOTCH FILTER
- WINDOWED FIR