ECE 498LV: Problem Set 3
Degree and Differentials

Released: Tuesday, February 13
Due: Thursday, February 22 (in class)

Be sure to show your work.

1. **[Second Degree]**
   Please complete Problem 8.3 in Newman’s textbook.

2. **[Third Degree]**
   Please complete Problem 8.4 in Newman’s textbook.

3. **[Fourth Degree]**
   Choose an unweighted and undirected real-world network with at least 100 nodes and produce a log-log plot of the survival function of the degree. Also use the statistical methodology of Clauset, Shalizi, and Newman (2009) to check whether or not your network has a degree distribution that is well-described as a power law. You may either use your own implementation or draw on the code from http://tuvalu.santafe.edu/~aaronc/powerlaws/.

   One possible, but quite large, network you can consider is a network of bitcoin transactions, which is available from https://github.com/ivan-brugere/Bitcoin-Transaction-Network-Extraction. Some relevant papers on the bitcoin network include Ober, Katzenbeisser, and Hamacher (2013), as well as Ermann, Frahm, and Shepelyansky (arXiv, 2017).

4. **[Differential Equations]**
   Consider a stable and causal LTI system where the input and output are related by the differential equation:
   \[
   \frac{d^2 y(t)}{dt^2} + 6 \frac{dy(t)}{dt} + 8y(t) = 2x(t).
   \]
   (a) Find the impulse response of the system.
   (b) What is the response of the system if \( x(t) = te^{-2t}1(t) \), where \( 1(\cdot) \) is the unit step function?

5. **[Difference Equations]**
   Consider a causal LTI system described by the difference equation:
   \[
   y[n] + \frac{1}{2} y[n-1] = x[n].
   \]
   (a) Determine the frequency response \( H(e^{j\omega}) \), i.e. the discrete-time Fourier transform of this system.
   (b) Determine the response of the system to the input \( x[n] = \delta[n] + \frac{1}{2} \delta[n-1] \) where \( \delta[\cdot] \) is the Kronecker delta function.

6. **[Fixed Points]**
   Consider the dynamical system:
   \[
   \frac{dx}{dt} = 14x - 2x^2 - xy,
   \]
   \[
   \frac{dy}{dt} = 16y - 2y^2 - xy.
   \]
   (a) Find the fixed points of this system.
   (b) Specify which (if any) of the fixed points are saddle points.
7. [More Fixed Points]
Consider the dynamical system:

\[
\begin{align*}
\frac{dx}{dt} &= 4x + 2y + 2x^2 - 3y^2, \\
\frac{dy}{dt} &= 4x - 3y + 7xy.
\end{align*}
\]

(a) Determine whether (0, 0) is a fixed point of the system.

(b) If (0, 0) is indeed a fixed point, determine whether it is attracting, repelling, or a saddle point. If it is not, you luck out with no part (b) to be completed.

8. [Project]
Please tell us your thoughts on your team and topic for the final project.