A path in a network is any sequence of vertices such that every consecutive pair of vertices is connected by an edge.

(In directed graph, respect arrows)

Length of path is number of edges traversed.

Consider simple graph with $A_{ij}$.

The product $A_{ik}A_{kj}$ is 1 if there is a length-2 path from $i$ to $j$ via $k$.

The total number $N_{ij}^{(2)}$ of paths of length 2 from $j$ to $i$ via any $k$ is

$$N_{ij}^{(2)} = \sum_{k=1}^{n} A_{ik}A_{kj} = [A^2]_{ij}.$$ 

For length-3 paths:

$$N_{ij}^{(3)} = \sum_{k,l=1}^{n} A_{ik}A_{kl}A_{lj} = [A^3]_{ij}.$$ 

Length-$r$:

$$N_{ij}^{(r)} = [A^r]_{ij}.$$ 

Special case is counting loops (paths that start and end at $i$):

$$[A^r]_{ii}$$

Total $\theta$ loops of length $r$ anywhere is the sum of our starting points

$$L_r = \sum_{i=1}^{n} [A^r]_{ii} = \text{Tr}(A^r)$$

but this counts separately loops of same vertices but different starting points. How many triangles are in a molecule?

A geodesic path, shortest path, is path between two vertices such no shorter path.

geodesic distance.
Closure: Path \(uvw\) is closed if third edge from \(uvw\) present

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\sqrt{ }\]

Clustering coefficient is fraction of paths of length 2 that are closed.

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C = \frac{\text{(number of closed paths of length 2)}}{\text{(number of paths of length 2)}}
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C = \frac{\text{(number of triangles)} \times 6}{\text{(number of paths of length 2)}}
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C = \frac{\text{(number of triangles)} \times 3}{\text{(number of connected triples)}}
\]

Networks are small world if large clustering coefficient and small average path length.