Parameters for undirected Markov random graphs

Density or edge ($\theta$)

Two-star ($\sigma_2$)

Three-star ($\sigma_3$)

Triangle ($\tau$)

And higher order star configurations
Definition 1 (Kronecker product of matrices) Given two matrices $A = [a_{i,j}]$ and $B$ of sizes $n \times m$ and $n' \times m'$ respectively, the Kronecker product matrix $C$ of dimensions $(n \cdot n') \times (m \cdot m')$ is given by

$$C = A \otimes B = \begin{pmatrix}
a_{1,1}B & a_{1,2}B & \cdots & a_{1,m}B \\
a_{2,1}B & a_{2,2}B & \cdots & a_{2,m}B \\
\vdots & \vdots & \ddots & \vdots \\
a_{n,1}B & a_{n,2}B & \cdots & a_{n,m}B
\end{pmatrix}.$$
(d) Adjacency matrix of $K_1$

\[
\begin{array}{ccc}
1 & 1 & 0 \\
1 & 1 & 1 \\
0 & 1 & 1 \\
\end{array}
\]

(e) Adjacency matrix of $K_2 = K_1 \otimes K_1$

\[
\begin{array}{ccc}
K_1 & K_1 & 0 \\
K_1 & K_1 & K_1 \\
0 & K_1 & K_1 \\
\end{array}
\]
(a) $K_3$ adjacency matrix ($27 \times 27$)

(b) $K_4$ adjacency matrix ($81 \times 81$)
Initiator $K_1$  

$K_1$ adjacency matrix

$K_3$ adjacency matrix
Theorem 5 (Multinomial degree distribution)  Kronecker graphs have multinomial degree distributions, for both in- and out-degrees.

Theorem 6 (Multinomial eigenvalue distribution)  The Kronecker graph $K_k$ has a multinomial distribution for its eigenvalues.

Theorem 7 (Multinomial eigenvector distribution)  The components of each eigenvector of the Kronecker graph $K_k$ follow a multinomial distribution.

Theorem 12  If $K_1$ has diameter $D$ and a self-loop on every node, then for every $k$, the graph $K_k$ also has diameter $D$. 
Definition 14 (Stochastic Kronecker graph) Let $P_1$ be a $N_1 \times N_1$ probability matrix: the value $\theta_{ij} \in P_1$ denotes the probability that edge $(i, j)$ is present, $\theta_{ij} \in [0, 1]$.

Then $k^{th}$ Kronecker power $P_1^{[k]} = P_k$, where each entry $p_{uv} \in P_k$ encodes the probability of an edge $(u, v)$.

To obtain a graph, an instance (or realization), $K = R(P_k)$ we include edge $(u, v)$ in $K$ with probability $p_{uv}, p_{uv} \in P_k$. 