ECE 498LV: Problem Set 5
Dynamical Systems

**Released:** Tuesday, February 14
**Due:** Tuesday, February 21 (in class)

Be sure to show your work.

1. **[Differential Equations]**
   Consider a stable and causal LTI system where the input and output are related by the differential equation:
   \[
   \frac{d^2 y(t)}{dt^2} + 6 \frac{dy(t)}{dt} + 8y(t) = 2x(t).
   \]
   (a) Find the impulse response of the system.
   (b) What is the response of the system if \(x(t) = te^{-2t}\mathbf{1}(t)\), where \(\mathbf{1}(\cdot)\) is the unit step function?

2. **[Differential Equations II]**
   Consider a causal and stable LTI system \(S\) with frequency response:
   \[
   H(j\omega) = \frac{j\omega + 4}{6 - \omega^2 + 5j\omega}.
   \]
   (a) Determine a differential equation relating the input \(x(t)\) and the output \(y(t)\) of \(S\).
   (b) Determine the impulse response \(h(t)\) of \(S\).
   (c) What is the output of \(S\) when the input is \(x(t) = e^{-4t}\mathbf{1}(t) - te^{-4t}\mathbf{1}(t)\).

3. **[Difference Equations]**
   Consider a causal LTI system described by the difference equation:
   \[
   y[n] + \frac{1}{2}y[n-1] = x[n].
   \]
   (a) Determine the frequency response \(H(e^{j\omega})\), i.e. the discrete-time Fourier transform of this system.
   (b) Determine the response of the system to the input \(x[n] = \delta[n] + \frac{1}{2}\delta[n-1]\) where \(\delta[\cdot]\) is the Kronecker delta function.

4. **[Fixed Points]**
   Consider the dynamical system:
   \[
   \frac{dx}{dt} = 14x - 2x^2 - xy,
   \]
   \[
   \frac{dy}{dt} = 16y - 2y^2 - xy.
   \]
   (a) Find the fixed points of this system.
   (b) Specify which (if any) of the fixed points are saddle points.

5. **[More Fixed Points]**
   Consider the dynamical system:
   \[
   \frac{dx}{dt} = 4x + 2y + 2x^2 - 3y^2,
   \]
   \[
   \frac{dy}{dt} = 4x - 3y + 7xy.
   \]
(a) Determine whether $(0, 0)$ is a fixed point of the system.

(b) If $(0, 0)$ is indeed a fixed point, determine whether it is attracting, repelling, or a saddle point. If it is not, you luck out with no part (b) to be completed.

6. **Project**

   Please tell us your initial thoughts on your team and idea for the final project.