ECE 488: Compound Semiconductors

M,W,F 11:00 – 11:50, 3013 ECEB
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Office Hours: Tuesday 13:00 – 14:00
Lecture 2: August 24th, 2016
Assignments

• Reading from “Compound Semiconductors and Devices – An Introduction”
  – Wed 8/24: §’s 1.1, 1.1.1, 1.1.2, 1.1.3
  – Fri 8/26: §’s 1.2, 1.2.1, 1.2.2, 1.2.3
  – Mon §’s 1.3, 1.4, 1.5
Today’s Agenda

• Questions/Announcements
• Quantum Mechanics for Engineers
# Tentative Schedule [1]

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**Guideline Only: Subject to Change**
Quantum Mechanics for Engineers
Basic Concepts

• **Quantization of Energy:** Emission or absorption of light takes place as indivisible units (photons) with zero mass and momentum $p$.

\[ E = \hbar \omega = h \nu = \frac{hc}{\lambda}, \quad p = \frac{h}{\lambda} \]

• **Wave-Particle Duality:** Electrons can be diffracted like electromagnetic waves, with the electron wavelength and momentum connected by:

\[ E = \frac{p^2}{2m}, \quad p = \hbar k = \hbar \frac{2\pi}{\lambda}, \quad \lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}} \]
Maxwell Equations for Homogeneous and Isotropic Materials

\[ \nabla \times E = -\frac{\partial B}{\partial t} \]
\[ \nabla \times H = \frac{\partial D}{\partial t} + J_{\text{cond}} = J_{\text{total}} \]
\[ \nabla \cdot D = \rho(x,y,z) \]
\[ \nabla \cdot B = 0 \]
\[ B = \mu H \]
\[ D(r,t) = \varepsilon E = \varepsilon_0 E + P \]
\[ J = \sigma E \]
\[ \nabla \cdot J = -\frac{\partial \rho}{\partial t} \]

\( E \): electric field
\( D \): electric displacement
\( B \): magnetic field
\( H \): magnetizing field
\( \varepsilon_s \): permittivity
\( \mu_o \): permeability
\( \rho \): total electric charge density
\( J_{\text{cond}} \): the conduction current density
\( P \): polarization density
\( \times \): curl operator
\( \cdot \): divergence operator
Derivation of Vector Wave Equation for Electromagnetic Waves

Maxwell’s Equations

Simplify for Free Space, Charge Free Region

Take Curl of $\nabla \times \mathbf{E} = (\nabla \times \nabla \times \mathbf{E})$

Apply Vector Identity:
$\nabla \times (\nabla \times \mathbf{E}) = \nabla \times \left( -\frac{\partial \mathbf{B}}{\partial t} \right)$

Simplify Expression to Show Final Form of Vector Wave Equation

Solve Vector Wave Equation to Give Expression for Electric Field Intensity of a $E_x$ Polarized Plane Wave Propagating in $z$-direction

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla \times \left( -\frac{\partial \mathbf{B}}{\partial t} \right)$$

$$\nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\mu \varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} - \frac{1}{\mu \varepsilon} \nabla^2 \mathbf{E} = 0$$

$$|E_x(z,t)| = A_E e^{i(kz-\omega t)}$$
**Electromagnetic Wave Propagation**

**Electromagnetic Wave Equation, Free Space:**

\[
\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2}
\]

\(\mu_0 \rightarrow \text{permeability}, \ \varepsilon_0 \rightarrow \text{permittivity}\)

**Solution:**

\[
|E(z, t)| = A_E e^{i(kz - \omega t)}
\]

\(A_E \rightarrow \text{amplitude, } \omega \rightarrow \text{oscillation frequency}\)

\(k \rightarrow \text{propagation constant}\)

\(k = \omega \sqrt{\mu_0 \varepsilon_0}\)
Particle Wave Function

\[ \psi(z, t) = Ae^{i(kz - \omega t)} \]

Wave Vector \( k \): \[ k = \frac{2\pi}{\lambda} \]

Frequency: \( \omega = 2\pi \nu \)

Energy: \( E = \hbar \omega = \hbar \nu \)

Momentum: \( p = \hbar k = \frac{\hbar}{\lambda} \)
The Schrödinger Wave Equation

$$\psi(z,t) = A \exp[i(kz - \omega t)]$$

$$\frac{d\psi}{dt} = A \exp[i(kz - \omega t)] \frac{d}{dt}[i(kz - \omega t)] = -i \omega \psi$$

$$i\hbar \frac{d\psi}{dt} = \hbar \omega \psi = E \psi \quad \text{and} \quad E(z) = \frac{\hbar^2 k^2}{2m} + V(z) = (KE + PE)$$

$$\frac{d\psi}{dz} = A \exp[i(kz - \omega t)] \frac{d}{dz}[i(kz - \omega t)] = ik \psi$$

$$\frac{d^2\psi}{dz^2} = -k^2 \psi$$

$$\Rightarrow \quad i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} (-k^2 \psi) + V(z) \psi = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial z^2} + V(z) \psi$$

1D: $$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi(z,t)}{\partial z^2} + V(z) \psi(z,t) = i\hbar \frac{\partial \psi(z,t)}{\partial t}$$

3D: $$-\frac{\hbar^2}{2m} \nabla^2 \psi(r,t) + V(r) \psi(r,t) = i\hbar \frac{\partial \psi(r,t)}{\partial t}$$
Analogy Between Electromagnetic Waves and Plane Matter Waves

**Electromagnetic Wave**

\[ |E_x(z,t)| = A_E e^{i(kz-\omega t)} \]

**Plane Matter Wave**

\[ \psi(z,t) = A e^{i(kz-\omega t)} \]

\[ k = \frac{2\pi}{\lambda} \quad \omega = 2\pi v \]

\[ k = \omega \sqrt{\mu \varepsilon} \]

\[ \lambda = \frac{hc}{E} \]

\[ k = \frac{p}{\hbar} \]

\[ \lambda = \frac{h}{\sqrt{2mE}} \]
Operators, Connection Rules

- $\psi(r,t)$ is a complex wave function associated with the particle.
- The classical energy of the system can be converted into the Schrödinger wave equation by associating certain operators.

$$E = \frac{p^2}{2m} + V \quad \iff \quad i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial z^2} + V(z)\psi$$

$$E \quad \iff \quad -\frac{\hbar}{i} \frac{\partial}{\partial t}$$

$$p \quad \iff \quad \frac{\hbar}{i} \nabla$$

$$p^2 \quad \iff \quad -\hbar^2 \nabla^2$$

- Connection rules or boundary conditions:
  - $\psi$ and $\nabla \psi$ are finite, continuous, and single valued.
- Probability:

$$\int \psi^* \psi \, dv = 1 \quad \text{for all space.}$$

It can be used to evaluate the constant of the wave function and the process is called ‘normalization’.
• Born Conditions:
  – Applied to ensure wave function is physically meaningful, and based upon the postulate that the square modulus of the wave function is the probability density of finding the system in a particular state.
  – The wave function must be square-integrable.
    • A consequence is that the wave function must tend to zero for infinite distances.
  – The wave function must be single valued.
  – The wave function must be continuous.
    • If the wave function is not continuous, the derivative would be infinite at a given point. Since the momentum operator is based on the first derivative, a discontinuous wave function would imply a point of infinite momentum.
  – The first derivative of the wave function must be continuous.
    • The energy operator is based on the second derivative of the wave function. If the first derivative is not continuous, the second derivative would have a point of infinite value, implying a point of infinite energy.

• In the case of non-physical problems, such as an infinite potential, the Born Conditions might be violated.
Dispersion Relationship

Dispersion of colors by a prism:

Isaac Newton (1672): 
A new theory about light and colours.

- The EM wave propagating in vacuum or air has a velocity of light speed which is independent of the wave frequency. Thus, air is a lossless or non-dispersive dielectric medium.

\[ v_p = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_o \varepsilon_o}} = c \]

- In other mediums, the wave velocity is a function of frequency. The dependence of the wave velocity on frequency is called dispersion. The dispersion of a wave function is described by the \( \omega-k \) relationship.
Wave Packets

- Consider two plane waves with equal amplitude but slightly different $\omega \pm \Delta \omega$ and $k \pm \Delta k$. The mixed wave function

$$\psi_1(z, t) = A \exp\left[i\left((k + \Delta k)z - (\omega + \Delta \omega)t\right)\right]$$

$$\psi_2(z, t) = A \exp\left[i\left((k - \Delta k)z - (\omega - \Delta \omega)t\right)\right]$$

becomes

$$\psi_1 + \psi_2 = 2A \cos(\Delta k z - \Delta \omega t) \exp[i(kz - \omega t)]$$
Phase and Group Velocity

- The wavelets inside the packet move along at the phase velocity.

\[ kz - \omega t = 0; \quad v_p = \frac{\omega}{k} = \frac{\hbar k}{2m} = \frac{p}{2m} \]

- The envelope moves at the group velocity.

\[ \Delta k z - \Delta \omega t = 0; \quad v_g = \frac{\Delta \omega}{\Delta k} \approx \frac{d\omega}{dk} = \frac{\hbar k}{m} = \frac{p}{m} = v(\text{classical}) \]

- For an electron:

\[ E = \frac{1}{2} m v^2 = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m} = \hbar \omega \]

\[ \omega = \left( \frac{\hbar}{2m} \right) k^2 \quad \text{non-linear} \]
Expectation Values

\[ \langle \alpha \rangle \equiv \int \psi^* \alpha \psi \, dv \]

For example, the 1D momentum expectation value is

\[ \langle p \rangle = \int \psi^* \left( \frac{\hbar}{i} \right) \frac{\partial \psi}{\partial z} \, dz \]
Agenda for Next Class

- Particle Scattering from a Potential Step
- Finite Wells
- Coulomb Well
Thank You!
Common Semiconductors

Fig. 21.4. Room-temperature bandgap energy versus lattice constant of common elemental and binary compound semiconductors.
Contact Information & Website

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Website:
https://courses.engr.illinois.edu/ece488/
Course Objectives
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• Develop a working knowledge of compound semiconductor materials and devices
• Provide a foundation for future advanced physical electronics courses
• Provide basic device knowledge to support a career in wireless communications or photonics
• Provide sufficient background such that you can begin to read and understand the literature on compound semiconductor materials and devices
Periodic Table of the Elements

For elements with no stable isotopes, the mass number of the isotope with the longest half-life is in parentheses.
Course Outline
Course Outline

- Review of semiconductor fundamentals
  - Elementary quantum mechanics
  - Atomic bonding and crystal structures
  - Electronic band structures of solids
- Compound semiconductor materials
  - Compound semiconductor crystals
  - Material technologies
- Properties of heterostructures
  - Basic heterostructure properties
  - Electrical properties of heterostructures
  - Optical properties of heterostructures
- Heterostructure devices
  - High-speed electronic devices
  - Semiconductor lasers
  - New device development
Course Description (Detailed)

- Review of quantum, mechanical basics including wave-particle duality, Schroedinger wave equation, one-dimensional free and bounded particles in quantum wells
- Introduction to compound semiconductor crystals, structural and electrical properties, free carrier concentration and Fermi-Dirac integral, III-V alloys
- Phase equilibrium, growth of bulk crystals and phase equilibrium, liquid phase epitaxy, vapor phase epitaxy, metalorganic chemical vapor deposition, molecular beam epitaxy
- Basic heterostructure properties, energy band alignment models, strain effect on the bandgap energies, abrupt p-n heterojunction in equilibrium, heterojunction under bias
- Electronic properties of real quantum wells, potential barrier and tunneling, superlattices and miniband, quantum wells in electric fields, modulation doping and two-dimensional electron gas
- Optical properties of dielectrics, absorption, radiative transitions - Einstein relations, stimulated emission, absorption and emission rates in semiconductors, transitions in degenerated semiconductors, nonradiative recombination processes
- Metal-semiconductor field-effect transistors, pseudomorphic high-electron mobility transistors, heterojunction bipolar transistors, transfer electron devices, resonant tunneling devices
- Photodetectors, solar cells, light-emitting diodes (LEDs), dielectric waveguide and heterostructure laser theories, quantum well lasers, distributed feedback lasers, vertical cavity surface emitting lasers
Prerequisites

- ECE340 or equivalent basic semiconductor course
- Physics background – Basic modern physics
- Math background – differential equations
Grading and Policies
## Grading

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<td>Homework &amp; Class Participation</td>
<td>30%</td>
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<tr>
<td>Quizzes (Dates Will be Announced)</td>
<td>10%</td>
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<tr>
<td>Mid-Term Exam</td>
<td>20%</td>
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<td>Final Exam</td>
<td>40%</td>
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**Homework:**
- Due 1 week after assigned, due in class, no late homework accepted

**Quizzes:**
- 2 quizzes, dates will be announced ahead of time, 20 minutes

**Exam(s):**
- Calculator allowed
- 8.5 X 11, hand-written, double-sided formula sheet

**Key Points:**
- Come to class
- Do your homework
- If you’re having problems attend office hours
Other Comments

• Ask questions if you have them
• Don’t miss quizzes, exams, or homework
• Turn off your cell phones
• No video recording or photography in class
• Include name and NetID on all documents turned in for credit
• Class notes (required) can be purchased from the ECE Supply Center
• Additional reading materials will be distributed in class or through the course website
• Reference for further reading (NOT required):
  • Solid state physics:
  • Semiconductor physics and devices:
    – S.L. Chuang, *Physics of Semiconductor Devices*
  • Quantum wells and heterostructures:
  • Compound semiconductor materials: