ECE 488: Compound Semiconductors

M,W,F 11:00 – 11:50, 3013 ECEB
Professor John Dallesasse
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Office Hours: Tuesday 13:00 – 14:00
Lecture 39: November 30th, 2016
Assignments

• Reading from “Compound Semiconductors and Devices – An Introduction”
  – Fri 11/18: §’s 9.7, 9.7.1, 9.7.2, 9.7.3
  – Fri 12/2: §’s 10.1, 10.1.1, 10.1.2, 10.1.3, 10.1.4, 10.1.5, 10.1.6
  – Mon 12/5: §’s 10.2, 10.2.1, 10.2.2, 10.3, 10.4, 10.4.1, 10.4.2, 10.4.3, 10.4.4, 10.4.5

• Homework: Posted Friday 11-18, Due Monday 12-5
Today’s Agenda

• Absorption and Stimulated Emission
• Optical Transitions Between Bands
• Fermi-Dirac Inversion Factor, Transparency, and Gain
• Non-Radiative Recombination Processes
| OCT 31: Optical Properties of Dielectric Media | NOV 2: Absorption in Semiconductors | NOV 4: Transitions Between Discrete States |
| NOV 7: Radiative and Non-Radiative Transitions Between Bands | NOV 9: Introduction to Heterojunction Devices, MESFETs | NOV 11: Modulation Doping |
| NOV 14: High Electron Mobility Transistors (HEMTs) | NOV 16: High Electron Mobility Transistors (HEMTs) | NOV 18: GaN High Electron Mobility Transistors; NOV 21-25: Thanksgiving |
| NOV 28: Heterojunction Bipolar Transistors (HBTs) | NOV 30: Heterojunction Bipolar Transistors | DEC 2: Heterostructure Lasers |
| DEC 5: Heterostructure Lasers | DEC 7: Photodiodes and Solar Cells; Last Lecture | FINAL EXAM: Per Registrar’s Office |

**Guideline Only: Subject to Change**
Absorption and Stimulated Emission
Absorption and Stimulated Emission

(a). Necessary conditions for ‘stimulated emission’:

It simply requires the downward radiative transition rate must exceed the upward photon absorption rate, \( r_{21} > r_{12} \),

\[
B_{21}f_2(1-f_1)P(E_{21}) > B_{12}f_1(1-f_2)P(E_{21}).
\]

Since \( B_{12} = B_{21} \), this condition becomes

\[
f_2(1-f_1) > f_1(1-f_2).
\]

It reduces to

\[
f_2 > f_1 \quad \text{(population inversion!)}
\]

\[
\exp[(F_2 - F_1)/kT] > \exp[(E_2 - E_1)/kT]
\]

or

\[
F_2 - F_1 > E_2 - E_1
\]

Separation of Fermi levels

Photon emission energy
**Relationships Involving Absorption Coefficient**

- **Net absorption rate, $R(\text{abs})$:**
  The net absorption rate is defined as the difference between the upward and downward transition rates. It also equals the product of the absorption coefficient and the photon flux ($= \text{photon density} \times \text{velocity}$).

  $$R(\text{abs}) = r_{12} - r_{21} \quad (\text{photons/eV-s-cm}^3)$$

  $$= B_{12}(f_1 - f_2)P(E_{21})$$

  $$= \alpha(E_{21})P(E_{21}) \nu_g \quad (1/cm)(\text{photon/eV-cm}^3)(\text{cm/s})$$

  $$\alpha(E_{21}) = \frac{R(\text{abs})}{P(E_{21})\nu_g} = \frac{B_{12}(f_1 - f_2)}{c/\bar{n}}$$

  $$\nu_g = \frac{d\omega}{dk} = \frac{d(h\omega)}{hdk} = (c/\bar{n})/\left[1 + (E/\bar{n})(dn/dE)\right] \approx \frac{c}{\bar{n}}$$

- **Net stimulated emission rate, $R(\text{st})$:**
  The difference between downward (stimulated) transition and upward (absorption) transition is defined as the net stimulated emission rate.

  $$R(\text{st}) = r_{21} - r_{12} \quad (\text{photons/eV-s-cm}^3)$$

  $$= B_{12}(f_2 - f_1)P(E_{21})$$

  $$\omega = 2\pi v = 2\pi \left(\frac{c}{\bar{n}} \frac{1}{\lambda}\right)$$

  $$= 2\pi \frac{c}{\bar{n}} \frac{k}{2\pi} = \frac{c}{\bar{n}} k$$
Stimulated and Spontaneous Emission Rates

\[ R(st) = -\left[ \frac{B_{12}(f_2 - f_1)}{c/n} \right] \left( \frac{c}{n} \right) P(E_{21}) = -\alpha(E_{21}) \left( \frac{8\pi \bar{m}^2}{\hbar^3 c^2} \right) \frac{E_{21}^2}{\exp(E_{21}/kT) - 1} \]

Since \([\exp(E_{21}/kT) - 1]^{-1}\) equals \(\langle n_p \rangle\), the photon distribution function, we can define a 'stimulated emission rate', \(r_{stm}(E_{21})\), which is independent of the photon distribution function.

\[ r_{stm}(E_{21}) = \frac{R(st)}{\langle n_p \rangle} = -\left( \frac{8\pi \bar{m}^2 E_{21}^2}{\hbar^3 c^2} \right) \alpha(E_{21}) \propto -\alpha(E_{21}) \]

- **Spontaneous emission rate, \(s_{21}\):**

\[ s_{21} = r_{sp}(E_{21}) = A_{21}f_2(1 - f_1) = \left( \frac{8\pi \bar{m}^3 E_{21}^2}{\hbar^3 c^3} \right) \left[ B_{21} \left( \frac{f_1 - f_2}{c/n} \right) \frac{f_2(1-f_1)}{(f_1 - f_2)} \right] \]

\[ r_{sp}(E_{21}) = \frac{8\pi \bar{m}^2 E_{21}^2}{\hbar^3 c^2} \exp\left[ E_{21} - (F_2 - F_1) \right]/kT - 1 = \frac{r_{stm}(E_{21})}{\exp\left[ E_{21} - (F_2 - F_1) \right]/kT - 1} \]

\[ r_{stm}(E_{21}) = r_{sp}(E_{21}) \left\{ \exp\left[ E_{21} - (F_2 - F_1) \right]/kT - 1 \right\} \]

- Both \(r_{stm}(E_{21})\) and \(r_{sp}(E_{21})\) are related to the absorption coefficient \(\alpha(E_{21})\), which can be determined experimentally.
Transition Probability $B_{12}$

It is essential to determine the absorption coefficient such that all transition rates of a two-level system can be obtained. For a discrete system, the absorption coefficient was derived as

$$\alpha(E_{21}) = \left(\frac{n}{c}\right)B_{12}(f_1 - f_2)$$

The only unknown term in this equation is $B_{12}$, the transition probability. $B_{12}$ relates the interactions of carriers in the solid and the electromagnetic radiation. It is derived quantum mechanically through *time-dependent perturbation theory* and has the following form.

$$B_{12} = \left(\frac{2\pi}{\hbar}\right)|M|^2 \delta(E_2 - E_1 - \hbar\omega)$$

This is called *Fermi’s Golden Rule*. $B_{12}$ is a non-zero value only when $E_2 - E_1 = \hbar\omega$. $|M|$ is the matrix element of the interaction wave functions.

$$M = \langle \Psi_1^*(r)|P|\Psi_2(r) \rangle = \int \Psi_1^*(r)|P|\Psi_2(r)d^3r = \frac{\hbar}{i} \int \nabla \Psi_1^*(r)\nabla \Psi_2(r)d^3r$$

$\Psi_1$ and $\Psi_2$ are the wave functions of the initial and final state, respectively, and $P$ is the momentum operator.
For band-to-band transition in a semiconductor, we have to consider contributions from all involved states in the conduction and valence bands separated by an energy $E = \hbar \omega$. The total absorption coefficient is the sum of absorption coefficient for all the energy levels separated by $\hbar \omega$.

\[
\alpha(\hbar \omega) = \int_{-\infty}^{\infty} \frac{B_{12}}{c/\hbar} (f_1 - f_2) D_v(E_v - E) D_c(E - E_c) dE
\]

\[
\propto \frac{1}{m^2 \overline{n}(\hbar \omega)} \int_{-\infty}^{\infty} D_v(E'') D_c(E') |M|^2 \left[ f_v(E'') - f_c(E') \right] dE
\]
In the above equation, the absorption coefficient depends on $D(E)$ and $|M|^2$. In degenerate semiconductors, the DOS becomes carrier concentration dependent and $|M|^2$ is for transitions between band tails deviating from the parabolic band approximation. However, we can use the simple non-degenerate approximation to understand the absorption and stimulated emission processes.

The right upper panel is the experimental absorption coefficient of GaAs at room temperature. Once the absorption data at all energy values are collected, one can calculate other optical transition rates. The lower panel shows the corresponding calculated spontaneous transition rate in comparison with the experimental photoluminescence spectrum.
Assume a condition where the valence band is completely full \( f_v(E) = 1 \), and the conduction band is completely empty \( f_c(E) = 0 \). This leads to an intrinsic absorption coefficient, \( \alpha_o(\hbar \omega) \).

\[
\alpha_o(\hbar \omega) = A^* \left( \hbar \omega - E_g \right)^{1/2}
\]

\[
A^* \approx \frac{e^2 (2m_r)^{3/2}}{m_e c \sinh^2} , \quad m_r = \left( \frac{1}{m_e^*} + \frac{1}{m_h^*} \right)^{-1}
\]

The constant \( A^* \) was derived by Bardeen et al. in 1954. For transitions in a semiconductor with partially filled conduction and valence bands, the absorption coefficient is obtained by multiplying the intrinsic value by \( (f_v - f_c) \), which we left out in deriving \( \alpha_o(\hbar \omega) \).

\[
\alpha(\hbar \omega) = \alpha_o(\hbar \omega) [f_v(k) - f_c(k)]
\]

The term \( [f_v(k) - f_c(k)] \) is the ‘Fermi-Dirac inversion factor’. \( f_c \) and \( f_v \) are the Fermi-Dirac functions for electrons in the conduction and valence bands, respectively. Under the condition of population inversion, \( f_v(k) - f_c(k) < 0 \), or \( f_v(k) < f_c(k) \). The absorption coefficient becomes negative and equals ‘gain coefficient’, \( g(\hbar \omega) \).

\[
g(\hbar \omega) = -\alpha(\hbar \omega)
\]
Fermi-Dirac Inversion Factor and Gain

Under population inversion condition,

\[
f_v(k) - f_c(k) = \frac{1}{1 + \exp\left[\frac{(E_v - F_c)/kT}{1 + \exp\left[\frac{(E_c - F_c)/kT}{kT}\right]}}\right] - \frac{1}{1 + \exp\left[\frac{(E_c - F_c)/kT}{kT}\right]}
\]

\[
= \frac{\exp\left[\frac{(E_c - F_c)/kT}{kT}\right] - \exp\left[\frac{(E_v - F_v)/kT}{kT}\right]}{\left[1 + \exp\left[\frac{(E_v - F_v)/kT}{kT}\right]\right]\left[1 + \exp\left[\frac{(E_c - F_c)/kT}{kT}\right]\right]} < 0
\]

\[\Rightarrow \exp\left[\frac{(E_c - F_c)/kT}{kT}\right] < \exp\left[\frac{(E_v - F_v)/kT}{kT}\right]
\]

\[F_c - F_v > E_c - E_v = \hbar\omega
\]

\[F_C \text{ and } F_V \text{ are the quasi-Fermi-level positions w.r.t. band edges.}
\]

Note- Under population inversion, the quasi Fermi levels move into conduction and valence band(tails) having a larger energy separation than the emission energy.
Joint DOS & Stimulated Emission

One can calculate the Fermi-Dirac inversion factor by evaluating the occupation probabilities over the energy $E = \hbar \omega - E_g$. The joint DOS has to be used for the stimulated emission process.

$$
\begin{align*}
E_c &= E_g + \frac{\hbar^2 k^2}{2m_e^*} \\
E_v &= -\frac{\hbar^2 k^2}{2m_h^*}
\end{align*}
$$

$$
\begin{align*}
(h\omega - E_g) &= \frac{\hbar^2 k^2}{2m_e^*} + \frac{\hbar^2 k^2}{2m_h^*} = \frac{\hbar^2 k^2}{2m_r^*} = \frac{1}{m_e^*} + \frac{1}{m_h^*} = \frac{\hbar^2 k^2}{2m_r} = E \\
E_c &= E_g + \frac{\hbar^2 k^2}{2m_r} \left( \frac{m_r}{m_e^*} \right) = E_g + \left( \frac{m_r}{m_e^*} \right) E; \quad E_v = -\left( \frac{m_r}{m_e^*} \right) E
\end{align*}
$$

$$
\begin{align*}
f_c(E) &= \frac{1}{1 + \exp\left( (E_c - F_c) / kT \right)} = \frac{1}{1 + \exp\left( \left[ E_g + \left( \frac{m_r}{m_e^*} \right) E - F_c \right] / kT \right)} \\
f_v(E) &= \frac{1}{1 + \exp\left( (E_v - F_v) / kT \right)} = \frac{1}{1 + \exp\left( -\left( \frac{m_r}{m_h^*} \right) E - F_v \right) / kT}\}
\end{align*}
$$

We can calculate the absorption/gain coefficient using the calculated $f(E)$ in

$$
\alpha(\hbar\omega) = \alpha_0(\hbar\omega) \left[ f_v(E) - f_c(E) \right]
$$
Agenda for Next Class

- Fermi-Dirac Inversion Factor, Transparency, and Gain
- Non-Radiative Recombination Processes
- Electronic Devices
Thank You!
## Final Exam

<table>
<thead>
<tr>
<th>Course</th>
<th>Section</th>
<th>CRN</th>
<th>Date</th>
<th>Day</th>
<th>Start Time</th>
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<th>Room</th>
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<tr>
<td>ECE 488</td>
<td>C</td>
<td>66375</td>
<td>12/12/2016</td>
<td>M</td>
<td>7:00 PM</td>
<td>10:00 PM</td>
<td>3017 Electrical &amp; Computer Eng Bldg</td>
<td>Extra Space</td>
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Periodic Table of the Elements

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<tr>
<th>Period</th>
<th>Group</th>
<th>Elements</th>
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<tr>
<td>1</td>
<td>I</td>
<td>H, He</td>
</tr>
<tr>
<td>2</td>
<td>II</td>
<td>Li, Be, B, C, N, O, F, Ne</td>
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<tr>
<td>3</td>
<td>III</td>
<td>Na, Mg, Al, Si, P, S, Cl, Ar</td>
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<tr>
<td>4</td>
<td>IV</td>
<td>K, Ca, Sc, Ti, V, Cr, Mn, Fe, Co, Ni, Cu, Zn, Ga, Ge, As, Se, Br, Kr</td>
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<tr>
<td>5</td>
<td>V</td>
<td>Rb, Sr, Y, Zr, Nb, Mo, Tc, Ru, Rh, Pd, Ag, Cd, In, Sn, Sb, Te, I, Xe</td>
</tr>
<tr>
<td>6</td>
<td>VI</td>
<td>Cs, Ba, La, Hf, Ta, W, Re, Os, Ir, Pt, Au, Hg, Tl, Pb, Bi, Po, At, Rn</td>
</tr>
</tbody>
</table>

For elements with no stable isotopes, the mass number of the isotope with the longest half-life is in parentheses.
Common Semiconductors

Fig. 21.4. Room-temperature bandgap energy versus lattice constant of common elemental and binary compound semiconductors.

“Italic” = indirect gap
“Roman” = direct gap
○ hexagonal structure
□ cubic structure
Contact Information & Website

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Website:
https://courses.engr.illinois.edu/ece488/
Course Objectives
Course Objectives

• Develop a working knowledge of compound semiconductor materials and devices
• Provide a foundation for future advanced physical electronics courses
• Provide basic device knowledge to support a career in wireless communications or photonics
• Provide sufficient background such that you can begin to read and understand the literature on compound semiconductor materials and devices
Course Outline
Course Outline

- Review of semiconductor fundamentals
  - Elementary quantum mechanics
  - Atomic bonding and crystal structures
  - Electronic band structures of solids
- Compound semiconductor materials
  - Compound semiconductor crystals
  - Material technologies
- Properties of heterostructures
  - Basic heterostructure properties
  - Electrical properties of heterostructures
  - Optical properties of heterostructures
- Heterostructure devices
  - High-speed electronic devices
  - Semiconductor lasers
  - New device development
Course Description (Detailed)

• Review of quantum, mechanical basics including wave-particle duality, Schroedinger wave equation, one-dimensional free and bounded particles in quantum wells
• Introduction to compound semiconductor crystals, structural and electrical properties, free carrier concentration and Fermi-Dirac integral, III-V alloys
• Phase equilibrium, growth of bulk crystals and phase equilibrium, liquid phase epitaxy, vapor phase epitaxy, metalorganic chemical vapor deposition, molecular beam epitaxy
• Basic heterostructure properties, energy band alignment models, strain effect on the bandgap energies, abrupt p-n heterojunction in equilibrium, heterojunction under bias
• Electronic properties of real quantum wells, potential barrier and tunneling, superlattices and miniband, quantum wells in electric fields, modulation doping and two-dimensional electron gas
• Optical properties of dielectrics, absorption, radiative transitions - Einstein relations, stimulated emission, absorption and emission rates in semiconductors, transitions in degenerated semiconductors, nonradiative recombination processes
• Metal-semiconductor field-effect transistors, pseudomorphic high-electron mobility transistors, heterojunction bipolar transistors, transfer electron devices, resonant tunneling devices
• Photodetectors, solar cells, light-emitting diodes (LEDs), dielectric waveguide and heterostructure laser theories, quantum well lasers, distributed feedback lasers, vertical cavity surface emitting lasers
Prerequisites

• ECE340 or equivalent basic semiconductor course
• Physics background – Basic modern physics
• Math background – differential equations
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<td>AUG 26</td>
<td>Infinite Square &amp; Triangle Wells</td>
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<td>AUG 29</td>
<td>Potential Steps, Coulomb Well (Hydrogen Atom), Atomic Bonding</td>
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<td>AUG 31</td>
<td>Crystal Structures, Diffraction</td>
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<td>SEP 2</td>
<td>Reciprocal Space, Diffraction Condition</td>
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<td>SEP 5</td>
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<td>SEP 7</td>
<td>The Brillouin Zone, Band Structures, Density of States</td>
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<td>SEP 9</td>
<td>Bloch Theorem, Empty Lattice Model</td>
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<td>SEP 12</td>
<td>Band Gaps</td>
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<tr>
<td>SEP 14</td>
<td>Kronig-Penny Model</td>
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<td>SEP 16</td>
<td>Effective Mass, Bloch Oscillations, Band Structure</td>
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<td>SEP 19</td>
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<td>SEP 23</td>
<td>Electrical Properties, Gap, Effective Mass, Mobility</td>
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**Guideline Only: Subject to Change**
## Tentative Schedule [2]

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<th>SEP 28: The Fermi Integral, Free Carrier Concentration, Surface States</th>
<th>SEP 30: III-V Semiconductor Lattice Constant and Bandgap</th>
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<td>OCT 3: III-N and Group IV Semiconductors</td>
<td>OCT 5: Crystal Growth, Phase Diagrams</td>
<td>OCT 7: Midterm Exam (Tentative)</td>
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<tr>
<td>OCT 10: Energy Band Alignment, Model-Solid Theory</td>
<td>OCT 12: Strained Layer Structures</td>
<td>OCT 14: Strain Effects on Band Edges</td>
</tr>
<tr>
<td>OCT 24: Realistic Finite Quantum Wells</td>
<td>OCT 26: Superlattices and Minibands</td>
<td>OCT 28: Heterostructures in Electric Fields and the Franz-Keldysh Effect</td>
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**Guideline Only: Subject to Change**
Grading and Policies
## Grading

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<tr>
<td>Homework &amp; Class Participation</td>
<td>30%</td>
</tr>
<tr>
<td>Quizzes (Dates Will be Announced)</td>
<td>10%</td>
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<tr>
<td>Mid-Term Exam</td>
<td>20%</td>
</tr>
<tr>
<td>Final Exam</td>
<td>40%</td>
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</table>

**Homework:**
- Due 1 week after assigned, due in class, no late homework accepted

**Quizzes:**
- 2 quizzes, dates will be announced ahead of time, 20 minutes

**Exam(s):**
- Calculator allowed
- 8.5 X 11, hand-written, double-sided formula sheet

**Key Points:**
- Come to class
- Do your homework
- If you’re having problems attend office hours
Other Comments

• Ask questions if you have them
• Don’t miss quizzes, exams, or homework
• Turn off your cell phones
• No video recording or photography in class
• Include name and NetID on all documents turned in for credit
• Class notes (required) can be purchased from the ECE Supply Center
• Additional reading materials will be distributed in class or through the course website
• Reference for further reading (NOT required):
  • Solid state physics:
  • Semiconductor physics and devices:
    – S.L. Chuang, *Physics of Semiconductor Devices*
  • Quantum wells and heterostructures:
  • Compound semiconductor materials: