ECE 488: Compound Semiconductors

M,W,F 11:00 – 11:50, 3013 ECEB
Professor John Dallesasse
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Office Hours: Tuesday 13:00 – 14:00
Lecture 38: November 28th, 2016
Assignments

• Reading from “Compound Semiconductors and Devices – An Introduction”
  – Fri 11/18: §’s 9.7, 9.7.1, 9.7.2, 9.7.3
  – Fri 12/2: §’s 10.1, 10.1.1, 10.1.2, 10.1.3, 10.1.4, 10.1.5, 10.1.6
  – Mon 12/5: §’s 10.2, 10.2.1, 10.2.2, 10.3, 10.4, 10.4.1, 10.4.2, 10.4.3, 10.4.4, 10.4.5

• Homework: Posted Friday 11-11, Due 11-18
• Next Homework: Posted Friday 11-18, Due Monday 12-5
Today’s Agenda

- Absorption Due to Isoelectronic Traps
- Radiative Transitions
- Einstein Relations
<table>
<thead>
<tr>
<th>OCT 31: Optical Properties of Dielectric Media</th>
<th>NOV 2: Absorption in Semiconductors</th>
<th>NOV 4: Transitions Between Discrete States **</th>
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<tr>
<td>NOV 7: Radiative and Non-Radiative Transitions Between Bands</td>
<td>NOV 9: Introduction to Heterojunction Devices, MESFETs</td>
<td>NOV 11: Modulation Doping</td>
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<td>NOV 14: High Electron Mobility Transistors (HEMTs)</td>
<td>NOV 16: High Electron Mobility Transistors (HEMTs)</td>
<td>NOV 18: GaN High Electron Mobility Transistors; NOV 21-25: Thanksgiving</td>
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<tr>
<td>NOV 28: Heterojunction Bipolar Transistors (HBTs)</td>
<td>NOV 30: Heterojunction Bipolar Transistors</td>
<td>DEC 2: Heterostructure Lasers</td>
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<tr>
<td>DEC 5: Heterostructure Lasers</td>
<td>DEC 7: Photodiodes and Solar Cells; Last Lecture</td>
<td>FINAL EXAM: Per Registrar’s Office</td>
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</table>

**Guideline Only: Subject to Change**
Indirect Transitions

Absorption Due to Isoelectronic Traps
Absorption Due to Isoelectronic Traps

- Nitrogen isoelectronic traps in GaP:
  - N replaces group-V atoms;
  - Large hydrostatic deformation of lattice and large electro-negativity difference between N and group-V element form a short-range potential well;
  - Allows the attraction of an electron in the well;
  - Exciton forms due to Coulomb force to attract a hole by trapped electron;
  - Electron is bound to the well in real space, which means the wave function of electron is diffused over a large $k$-space ($k \propto 1/r$).
  - Enhancement of direct transition probability near $k = 0$. 

\[ \Delta x \cdot \Delta p \geq \frac{\hbar}{2} \]
Isoelectronic Traps (2)

- By adding N-isoelectronic traps, the wave function probability is strongly enhanced near $k = 0$.
- Lightly doped GaP:N PL spectra at 2K. The zero phonon exciton transition lines A and B dominate the transitions.
Isoelectronic Traps (3)

- PL spectra of highly doped \(2 \times 10^{18} \text{ cm}^{-3}\) GaP:N at 2K.
- Interaction between neighboring isoelectronic traps forming new sets of energy levels such as \(\text{NN}_1, \text{NN}_2\), etc. Together, they form deeper well to attract electrons and forming excitons.
Isoelectronic Traps (4)

- GaP:N at very high doping level of $10^{19}$ cm$^{-3}$.
- The NN$_1$ transition associates with the nearest paired isoelectronic centers, which has the deepest potential well and strongest binding force.
Radiative Transitions
Photon Density Distributions

The interactions of light and free-carriers in a semiconductor, including absorption, spontaneous emission, and stimulated emission, can be described by Einstein’s relations. These transition processes are connected through the ‘photon density distribution’, $P(E)$, which describes the number of available photons participating in the interaction per unit energy and per unit volume ($#/eV\cdot cm^3$). Similar to the calculation of the electron distribution, $P(E)$ is determined by the product of the number of allowed photon states or photon density of states, $dD_p(E)$, and the probability that a photon will occupy that state determined by the photon distribution function, $\langle n_p \rangle$.

$$P(E) \sim dD_p(E) \cdot \langle n_p \rangle$$
Photon Density of States: $D_p(E)$

$D_p(E)$ can be derived just like the DOS of carriers in a crystal with a dimension $L \gg \lambda$. The particle-like nature of the electromagnetic radiation is represented by photons with energy $E = \hbar \omega = |p|c$ and $p = \hbar k$. The allowed wave vectors are (See CH.3, discussions on DOS.)

$$k_i = \frac{2\pi m_i}{L} \quad i = x, y, z$$

where $m_i = 1, 2, 3 \ldots$ Therefore, each allowed state occupies $(2\pi/L)^3$ in $k$-space. The photon density of states within a thin spherical shell in $k$-space, with a thickness of $dk$, is expressed as

$$dD_p(E) = 2 \times \left[ \frac{4\pi k^2 dk}{(2\pi/L)^3} \right] \frac{1}{L^3} = \frac{k^2 dk}{\pi^2}$$

The factor of 2 counts two different polarizations, $TE$ and $TM$, for a photon. In a dielectric medium with a refractive index $\bar{n}$, the wave vector is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{c/(\bar{n}v)} = \frac{2\pi \bar{n}}{c} \left( \frac{E}{\hbar} \right) \quad (E = h\nu)$$

$$dk = \frac{2\pi}{ch} (\bar{n}dE + Ed\bar{n}) = \frac{2\pi \bar{n}}{ch} \left( 1 + \frac{E}{\bar{n}} \frac{d\bar{n}}{dE} \right) dE$$

$$\therefore \quad dD_p(E) = \frac{8\pi \bar{n}^3 E^2}{h^3 c^3} \left( 1 + \frac{E}{\bar{n}} \frac{d\bar{n}}{dE} \right) dE \quad \text{(\# states/m}^3)$$
(b). Photon distribution function, $\langle n_p \rangle$:

Since photons are indistinguishable, identical particles, and do not follow Pauli’s exclusion principle, $\langle n_p \rangle$ is given by the Bose-Einstein distribution law.

$$\langle n_p \rangle = \frac{1}{\exp(E/kT) - 1} \quad \text{(\# photons/state)}$$

(c). Photon density distribution, $P(E)$:

$$P(E) = \frac{dD_p(E) \times \langle n_p \rangle}{dE}$$

$$= \frac{8\pi \bar{n}^3 E^2}{h^3 c^3} \cdot \frac{1 + \frac{E}{\bar{n}} \frac{dn}{dE}}{\exp(E/kT) - 1} \quad \text{(\# photons/eV - m$^3$)}$$

In general, $dn/dE$ is a smooth curve with finite variation and \(1 + \frac{E}{\bar{n}} \frac{dn}{dE}\) $\approx$ 1.

$$P(E) \approx \frac{8\pi \bar{n}^3 E^2}{h^3 c^3 \left[\exp(E/kT) - 1\right]}$$
Einstein Relations

The absorption of a photon of energy $E_{21} = \hbar \omega \geq E_g$ in a semiconductor results in an electron transition from a state $E_1$ in the valence band to a state $E_2$ in the conduction band. The transition rate between these two discrete states depends on:

- $B_{12}$ - transition probability;
- $f_1$ - probability that the state $E_1$ contains an electron;
- $(1 - f_2)$ - probability that the state $E_2$ is empty;
- $P(E_{21})$ - photon density of energy $E_{21}$. 
Absorption & Emission

(a). Upward transition rate, \( r_{12} \):

\[
r_{12} = B_{12} f_1 (1 - f_2) P(E_{21})
\]

\[
f_1 = \frac{1}{1 + \exp (E_1 - F_1)/kT}
\]

and

\[
f_2 = \frac{1}{1 + \exp (E_2 - F_2)/kT}
\]

\( F_1 \) and \( F_2 \) are quasi-Fermi levels.

(b). Stimulated emission rate, \( r_{21} \):

\[
r_{21} = B_{21} f_2 (1 - f_1) P(E_{21})
\]

where \( B_{21} \) is the downward transition transition probability.

(c). Spontaneous emission rate, \( s_{21} \):

Electrons can spontaneously return to \( E_1 \) without interacting with the radiation field and independent of \( P(E_{21}) \). However, its transition probability has a different value \( A_{21} \).

\[
s_{21} = A_{21} f_2 (1 - f_1)
\]
Derivation of Einstein Relations

Under thermal equilibrium condition,

\[
\begin{align*}
  r_{12} &= r_{21} + s_{21} \\
  F_1 &= F_2 = F \\
  E_2 - E_1 &= E_{21}
\end{align*}
\]

\[
B_{12} f_1 (1 - f_2) P(E_{21}) = B_{21} f_2 (1 - f_1) P(E_{21}) + A_{21} f_2 (1 - f_1)
\]

\[
\therefore \quad P(E_{21}) = \frac{A_{21} f_2 (1 - f_1)}{B_{12} f_1 (1 - f_2) - B_{21} f_2 (1 - f_1)} = \frac{A_{21}}{B_{12} \left[\frac{f_1 (1 - f_2)}{f_2 (1 - f_1)}\right] - B_{21}}
\]

\[
\frac{f_1 (1 - f_2)}{f_2 (1 - f_1)} = \frac{(1 - f_2) / f_2}{(1 - f_1) / f_1} = \frac{1/f_2 - 1}{1/f_1 - 1} = \frac{1 + \exp[(E_2 - F)/kT]}{1 + \exp[(E_1 - F)/kT]} - 1 = \exp[(E_2 - E_1)/kT] = \exp(E_{21}/kT)
\]

Therefore,

\[
P(E_{21}) = \frac{A_{21}}{B_{12} \exp(E_{21}/kT) - B_{21}} = \frac{8\pi \bar{n}^3 E_{21}^2}{h^3 c^3 \left[\exp(E_{21}/kT) - 1\right]}
\]

\[
\Rightarrow \quad A_{21} \left[\exp(E_{21}/kT) - 1\right] = \frac{8\pi \bar{n}^3 E_{21}^2}{h^3 c^3} \left[ B_{12} \exp(E_{21}/kT) - B_{21} \right]
\]
Einstein Relations

Comparing the temperature-independent and temperature-dependent terms on each side, we reach the Einstein’s relations:

\[
A_{21} = \left( \frac{8\pi \bar{n}^3 E_{21}^2}{h^3 c^3} \right) B_{21} \quad \text{and} \quad B_{12} = B_{21}
\]

These relations indicate that the spontaneous emission rate is related to the absorption rate and the stimulated emission rate. The absorption upward transition probability \(B_{12}\) equals the stimulated downward transition probability \(B_{21}\).

\[
A_{21} \left[ \exp \left( \frac{E_{21}}{kT} \right) - 1 \right] = 8\pi \bar{n}^3 \frac{E_{21}^2}{h^3 c^3} \left[ B_{12} \exp \left( \frac{E_{21}}{kT} \right) - B_{21} \right]
\]

\[
A_{21} e^{E_{21}/kT} - A_{21} = 8\pi \bar{n}^3 \frac{E_{21}^2}{h^3 c^3} B_{12} e^{E_{21}/kT} - 8\pi \bar{n}^3 \frac{E_{21}^2}{h^3 c^3} B_{21}
\]

so:

\[
A_{21} = \frac{8\pi \bar{n}^3 E_{21}^2}{h^3 c^3} B_{12}
\]

\[
\frac{8\pi \bar{n}^3 E_{21}^2}{h^3 c^3} B_{21} e^{E_{21}/kT} - \frac{8\pi \bar{n}^3 E_{21}^2}{h^3 c^3} B_{21} = \frac{8\pi \bar{n}^3 E_{21}^2}{h^3 c^3} B_{12} e^{E_{21}/kT} - \frac{8\pi \bar{n}^3 E_{21}^2}{h^3 c^3} B_{21}
\]


Agenda for Next Class

- Relationship Between Absorption, Stimulated Emission, and Spontaneous Emission
- Transitions Between Bands
- Optical Absorption in Quantum Wells
Thank You!
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<thead>
<tr>
<th>Course</th>
<th>Section</th>
<th>CRN</th>
<th>Date</th>
<th>Day</th>
<th>Start Time</th>
<th>End Time</th>
<th>Room</th>
<th>Exam Type</th>
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<tr>
<td>ECE 488</td>
<td>C</td>
<td>66375</td>
<td>12/12/2016</td>
<td>M</td>
<td>7:00 PM</td>
<td>10:00 PM</td>
<td>3017 Electrical &amp; Computer Eng Bldg</td>
<td>Extra Space</td>
</tr>
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Periodic Table of the Elements

For elements with no stable isotopes, the mass number of the isotope with the longest half-life is in parentheses.
Common Semiconductors

![Graph showing bandgap energy (E_g) vs. lattice constant (a) for various semiconductors.]

- **“Italics”** = indirect gap
- **“Roman”** = direct gap
- ◆ hexagonal structure
- □ cubic structure

Fig. 21.4. Room-temperature bandgap energy versus lattice constant of common elemental and binary compound semiconductors.

E. F. Schubert

Light-Emitting Diodes (Cambridge Univ. Press)

www.LightEmittingDiodes.org
Contact Information & Website

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Website:
https://courses.engr.illinois.edu/ece488/
Course Objectives
Course Objectives

- Develop a working knowledge of compound semiconductor materials and devices
- Provide a foundation for future advanced physical electronics courses
- Provide basic device knowledge to support a career in wireless communications or photonics
- Provide sufficient background such that you can begin to read and understand the literature on compound semiconductor materials and devices
Course Outline
Course Outline

- Review of semiconductor fundamentals
  - Elementary quantum mechanics
  - Atomic bonding and crystal structures
  - Electronic band structures of solids
- Compound semiconductor materials
  - Compound semiconductor crystals
  - Material technologies
- Properties of heterostructures
  - Basic heterostructure properties
  - Electrical properties of heterostructures
  - Optical properties of heterostructures
- Heterostructure devices
  - High-speed electronic devices
  - Semiconductor lasers
  - New device development
Course Description (Detailed)

- Review of quantum, mechanical basics including wave-particle duality, Schroedinger wave equation, one-dimensional free and bounded particles in quantum wells
- Introduction to compound semiconductor crystals, structural and electrical properties, free carrier concentration and Fermi-Dirac integral, III-V alloys
- Phase equilibrium, growth of bulk crystals and phase equilibrium, liquid phase epitaxy, vapor phase epitaxy, metalorganic chemical vapor deposition, molecular beam epitaxy
- Basic heterostructure properties, energy band alignment models, strain effect on the bandgap energies, abrupt p-n heterojunction in equilibrium, heterojunction under bias
- Electronic properties of real quantum wells, potential barrier and tunneling, superlattices and miniband, quantum wells in electric fields, modulation doping and two-dimensional electron gas
- Optical properties of dielectrics, absorption, radiative transitions - Einstein relations, stimulated emission, absorption and emission rates in semiconductors, transitions in degenerated semiconductors, nonradiative recombination processes
- Metal-semiconductor field-effect transistors, pseudomorphic high-electron mobility transistors, heterojunction bipolar transistors, transfer electron devices, resonant tunneling devices
- Photodetectors, solar cells, light-emitting diodes (LEDs), dielectric waveguide and heterostructure laser theories, quantum well lasers, distributed feedback lasers, vertical cavity surface emitting lasers
Prerequisites

• ECE340 or equivalent basic semiconductor course
• Physics background – Basic modern physics
• Math background – differential equations
# Tentative Schedule [1]

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<tr>
<th>AUG 22: Introductions, Objectives, Class Outline, Policies</th>
<th>AUG 24: Motivation, Intro to Quantum Theory</th>
<th>AUG 26: Infinite Square &amp; Triangle Wells</th>
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<tr>
<td>AUG 29: Potential Steps, Coulomb Well (Hydrogen Atom), Atomic Bonding</td>
<td>AUG 31: Crystal Structures, Diffraction</td>
<td>SEP 2: Reciprocal Space, Diffraction Condition</td>
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<tr>
<td>SEP 5: LABOR DAY NO CLASS</td>
<td>SEP 7: The Brillouin Zone, Band Structures, Density of States</td>
<td>SEP 9: Bloch Theorem, Empty Lattice Model</td>
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<tr>
<td>SEP 12: Band Gaps</td>
<td>SEP 14: Kronig-Penny Model</td>
<td>SEP 16: Effective Mass, Bloch Oscillations, Band Structure</td>
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**Guideline Only: Subject to Change**
### Tentative Schedule [2]

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<tr>
<th>SEP 26: Doping and Deep Levels</th>
<th>SEP 28: The Fermi Integral, Free Carrier Concentration, Surface States</th>
<th>SEP 30: III-V Semiconductor Lattice Constant and Bandgap</th>
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<td>OCT 3: III-N and Group IV Semiconductors</td>
<td>OCT 5: Crystal Growth, Phase Diagrams</td>
<td>OCT 7: Midterm Exam (Tentative)</td>
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<tr>
<td>OCT 10: Energy Band Alignment, Model-Solid Theory</td>
<td>OCT 12: Strained Layer Structures</td>
<td>OCT 14: Strain Effects on Band Edges</td>
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<tr>
<td>OCT 24: Realistic Finite Quantum Wells</td>
<td>OCT 26: Superlattices and Minibands</td>
<td>OCT 28: Heterostructures in Electric Fields and the Franz-Keldysh Effect</td>
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**Guideline Only: Subject to Change**
Grading and Policies
# Grading

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<th>Grading Category</th>
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<td>Homework &amp; Class Participation</td>
<td>30%</td>
</tr>
<tr>
<td>Quizzes (Dates Will be Announced)</td>
<td>10%</td>
</tr>
<tr>
<td>Mid-Term Exam</td>
<td>20%</td>
</tr>
<tr>
<td>Final Exam</td>
<td>40%</td>
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</table>

**Homework:**
- Due 1 week after assigned, due in class, no late homework accepted

**Quizzes:**
- 2 quizzes, dates will be announced ahead of time, 20 minutes

**Exam(s):**
- Calculator allowed
- 8.5 X 11, hand-written, double-sided formula sheet

**Key Points:**
- Come to class
- Do your homework
- If you’re having problems attend office hours
Other Comments

• Ask questions if you have them
• Don’t miss quizzes, exams, or homework
• Turn off your cell phones
• No video recording or photography in class
• Include name and NetID on all documents turned in for credit
Class notes (required) can be purchased from the ECE Supply Center
Additional reading materials will be distributed in class or through the course website
Reference for further reading (NOT required):

Solid state physics:
  - C. Kittel, *Introduction to solid state physics* (any edition), John Wiley

Semiconductor physics and devices:
  - S.L. Chuang, *Physics of Semiconductor Devices*

Quantum wells and heterostructures:

Compound semiconductor materials: